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THE MONIST

THE RELATIONAL ELEMENT IN MONISM.

I.

IN the ever-changing panorama which the world offers to the contemplation of sentient beings, two characters have never failed to arouse the interest of the human mind, to provoke it to the activities of philosophic thought—the manner in which objects are related to each other, and the nature of the objects themselves. Being under necessity of adapting himself to it, man looks to the external world for the materials of an organized intellectual awareness to its twofold aspect of extension, or so-called "space," of succession, or so-called "time." Among the most important of the conditions of this awareness, two may be mentioned—on the one hand man's local position in nature and limited power of movement within it, restricting him to a mere fragment of its vast spatial extent; on the other hand, that serial character of the mind process which, yielding direct glimpses in such narrow area of no more than momentary aspects of the nature there accessible to him, confines him largely to the appearances yielded by objects in the present stage of their cycle of change. The conditions of man's existence thus combine with the limitations of sense to favor incomplete and therefore erroneous views of nature—lead man, that is to say, to treat the local environment with which he is familiar as if it were the whole environment; to regard the present stage of the objects with which he is acquainted as if it constituted all the stages of such objects; in a word, to confound each sense-image with the object which it merely represents, and thus to make the temporary and local aspects

of the world around him do duty in his intellectual processes for nature in its totality.

Now the actual mechanism of these limitations—given man's local position in "space"—has always been the sense-image; and as the sense-image restricts us to single objects at a time, as well as to the present aspects of such objects, it makes our power to realize what objects are as wholes in "time" and "space" depend on that supplementary intellectual process by which the mind segregates and unifies all the objective aspects of the environment which sense-images yield. But as this intellectual process, this ordering of the sense-images by the intellect, this totalizing of the fragmentary elements of sense, matures late in the history of the race, the early period of man's acquaintance with nature is one in which he is not only restricted to little more than sense-image knowledge of it, but is compelled to view objects as dynamically isolated from, as out of relation to each other. For as sense-knowledge contains no evidence, apart from the intellectual process, of the way in which objects are intimately and fundamentally related to each other, of how they are connected by action and reaction, it compels the primitive mind to treat each of them as having absolute instead of contingent existence, as possessing in and of itself all the qualities or properties which it manifests, as being self-sourced instead of system-sourced.

There has undoubtedly been advance on this primitive treatment of objects as if they existed independently of each other; considerable progress has even been made towards the realization of their dynamic interrelatedness. For sense-images are merely the materials of nature knowledge; in the earliest stages of its development, the human mind was always at work upon this sense-material, classifying, segregating, unifying it under the stress of an impulsion to self-maintenance that demanded of the organism an increasingly accurate adjustment of itself to environment. Yet it is only in the modern stage that this intellectual process has attained anything like considerable development. Releasing the mind from more or less dependence on the present aspect of an object, it has enabled us to realize the object as constituted of all its aspects—of all the changes through which it passes—and thus to deal with it intel-

lectually as a whole in time; by simultaneously revealing interaction between the individual object and other objects, it has enabled us to recognize that objects are not isolated from each other, but dynamically interrelated and interdetermining.

That the doctrine of "evolution," so called, should have formed the first great step towards the completion of nature knowledge becomes explicable only when regard is had to the fact that the relations earliest revealed to the mind are always those of change. As the "time" aspects of objects are aspects of difference; as it is the procession of an object from one stage of its existence to another and not its continuing existence, which most excites our attention—it was inevitable that when the human mind began to reach out for the deeper and wider acquaintance with objects which is to be gained only from a knowledge of their connection with each other, it should have been overwhelmingly impressed, not with the fact of their relation in "space," but with the phenomena of their relation in "time." And if it be asked why the mind has simultaneously fallen short in the equally necessary process of realizing objects as totalities in their extension aspect, the reply is that the field is here not one of differences that shock the consciousness because they embody change, but of resemblances cognizable by the intellect rather than by the senses—that the deeper extension aspects of existence, being aspects of the fundamental, the universal, the permanent, manifest themselves, not as differences, but as likenesses, and long resisting the solvent of the sense knowledge, yield only to the intellectual process, thus coming late into nature knowledge as one of its most mature and advanced stages. The result of this failure to complete our succession or "evolutionary" knowledge of objects with an equally radical insight into the extension aspect of nature has been, so far as the fundamental method of its thought is concerned, to retain science in a stage which has advanced little beyond that represented by the mind of primitive man. There is undoubted recognition of the fact that objects are interrelated by action, yet the action is viewed independently of the nature of the objects which act. Alike in regard to their motion and nature, objects are regarded, not as contingent, but as absolute existences — not as

system-sourced, but as self-sourced—not as constitutively, but as adventitiously related.

II.

Let us now see what this absolute view of objects must signify for nature knowledge, even when pervaded and permeated here and there by glimpses of the larger relational view. Observe first how it diminishes interest in and retards discovery of the relations which exist between the different phases of objects in both "time" and "space." For if an object as extended possess its properties absolutely, no new knowledge of it can be gained by inquiring into the properties of other objects; if, on the other hand, it possess each "time" phase of its existence absolutely, nothing can be gained by inquiring into the nature of the other phases. Whence it follows that, were we to accept the absolute view of objects—the view that they are self-sourced rather than system-sourced—it could only be a waste of effort to seek for the individual being the explanation yielded by the human society, to consider the cell in the light of the community of cells, to study life in general in order to obtain accurate knowledge of some special class of organisms, to rely on the inorganic for the characters of the organic, to resort to the universe for the illumination of the unit. In the view that each object possesses its powers independently of all other objects, the search for cause, which has done so much for the advance of nature knowledge, dwindles to a mere quest after relation between states of consciousness until the organism comes to be regarded as in no way necessarily dependent for such states upon any real, upon any actual universe which can be conceived of as existing outside. We find this uncertainty even in Kant; the very belief of Hume, still lingering in the philosophical aspect of nature study, to the effect that our supposed knowledge of cause is wholly subjective—that what we call causal relation is merely the habitual succession of states of consciousness—is itself due to the inadequate recognition of the relation which exists between organism and environment—failure to view conscious states as necessarily related to objects outside the organism themselves interrelated—failure also to regard objective relations as constitutive instead of merely adventitious.

We may begin our consideration of the retarding effects of the absolute view, here contrasted with the relational view of nature, by noting the almost complete indifference which science has thus far shown towards the problem of the relation of substance—of the world of ether and matter—to so-called "space"; and it has shown this indifference in the supposition that the problem is one of no importance for the practical interests of nature investigation. Meanwhile, it has tacitly accepted the view that "space," as a containing capacity for objects which would remain even were objects annihilated, is a real entity, an existence *sui generis*. Says Tyndall, in his paper on *The Constitution of Nature*:

"Though compelled to think of space as unbounded, there is no mental necessity appealing to us to think of it either as filled or empty; whether it is so or not must be decided by experiment and observation....If the ether have a boundary, masses of ponderable matter might be conceived to exist beyond it, but they emit no light....As far as our knowledge of space extends, we are to conceive of it as the holder of the luminiferous ether through which are interspersed, at enormous distances apart, the ponderous nuclei of the stars."

The result of specialism thus unfounded in the intellectual conditions of nature knowledge has been to delay our realization of the universe as a unity, since the assumption of an entity called "space," which existence merely "occupies," compels us to compromise in our minds between two ideas—the conception of a universe of objects which "fills" space and is therefore unending, and the conception of a cosmic system which, being limited, is bounded in every direction by "universal space." If we accept the former view, the notion of definite characters—definite amounts of power and of the passing over of power into movement—disappears in the thought of indefinite spatial vastness, or is reduced to an *impasse* of contradiction between characters which, though mentally conceivable, are intellectually irreconcilable; if we accept the view of a limited cosmos locally situated in "universal space," we raise the question of the possibility of other systems, and with it that problem of the relation between them which, unless affirmatively resolved, with the result of irreconcilable contradiction, brings the thought of cosmic unity to an end: as the term "universe," implying "one-

ness," signifies all-inclusive totality, it becomes a misnomer when an attempt is made to utilize it for the pseudo-conception of a plurality of "universes" separated from each other by tracts of unoccupied emptiness. That the thought of an existence vacuum which objects merely "occupy" has otherwise retarded nature knowledge is well seen in the assumption, frequently met with in scientific treatises, that heat may be dissipated from "our universe" into "space," as well as in the historic confusion of "empty space" with the ether system, and in the contradictory assumptions of popular science regarding what are called "the confines of the universe," the "limits of our system," the "material universe," etc.

III.

In now passing from the most general to the more concrete of the nature problems, it will help us if we bear in mind that the absolute view of objects—the view that each exists independently of all the rest, and that the relation between them is adventitious, not constitutive—ignores three principles of the profoundest import for nature study. These principles may be stated as follows:

1. The Principle of Likeness. Objects which, however they may differ in superficial characters, resemble each other in fundamental characters, belong to the same system, and must be interpreted in the same way.

2. The Principle of Derivation. Wherever in local, superficial, transitory aspects of nature we find characters that demand explanation, we are bound to seek that explanation not in other local, superficial, transitory aspects of existence, but in its permanent, fundamental and universal characters.

3. The Principle of Universality. True explanations of nature cannot be isolated explanations, explanations *sui generis,* explanations *ad rem,* but must be explanations which are themselves interrelated. In other words, all great laws which are to formulate or describe nature processes must be laws that are primordially valid for the universal, the fundamental, the permanent characters of the cosmic process, before being derivatively valid for its local, superficial, and transitory characters. The explanatory range of every

true natural law, that is to say, must resemble, to use a homely illustration, the range of a through train which reaches the subsidiary, suburban, outlying stations only because it has begun its trip in the great metropolis.

All three principles assume and demand the thought of the all-inclusive unity of the cosmos. The first, aimed at the failure to realize cosmic unity, preserves us from separating in our thought great classes of phenomena that belong together; the second saves us from the temptation, so strongly held out by sense knowledge and our local position in the universe, to reverse true principles of interpretation by employing the local and immediate for the elucidation of the permanent and the universal; the third emphasizes the orderly filiation of all explanations of nature phenomena by referring such phenomena to their ultimate source in the substance system. Accepting all three, as we are compelled to do, then it follows—from the principle of likeness, that matter and ether, being connected by action and reaction, are of fundamentally the same nature—from the principle of derivation, that we must interpret the mind by the body, and not the body by the mind; the individual by society, and not society by the individual; the cell by the organism, not the organism by the cell; the organic by the inorganic, and not the inorganic by the organic; matter by substance, not substance by matter; the unit by the universe, not the universe by the unit—from the principle of universality, that whatever explanations are reached for vital, for chemical, for electro-magnetic, for gravitative phenomena, must be interrelated explanations, and that the causal determinations which result in organic activity and structure cannot be other than derivative forms of the causal determinations out of which inorganic activities and structures arise.

Now all these principles are more or less negatived or ignored in the absolute attitude towards nature—in the tacit acceptance by our so-called "scientific method" of the view that objects, possessing their characters absolutely, are related to each other not constitutively, but only adventitiously—in views, that is to say, which really go together, for if an object be self-sourced, then any relation between it and other objects must be adventitious, while if the object

be system-sourced, then any relation between it and other objects must be constitutive. Taking our first example from the most important of the concrete nature problems, we may easily recognize that, by ignoring the principle of likeness, the physicist need not assume that there is any relation other than an adventitious one between matter and ether; while, by denying the principle of derivation, he may even seek to explain ether from matter—the universal from the merely local element, with the result of one or other of the familiar comparisons in which physicists liken ether to jelly, wax, or steel. Well-nigh all the difficulties thus far met with by the human mind in trying to realize the nature of ether have been difficulties resulting from the effort to approximate it to the nature of matter. Le Sage used material corpuscles to elucidate gravitative action; Newton's theory of light had an analogous basis in matter; heat was long traced to material elements which could be mingled with and obtained from the molecules of objects; the early "fluid" hypothesis of electricity anticipated, in its material character, certain modern methods of accounting for the luminous manifestations of electro-magnetic phenomena; it is still, in our own time found possible to formulate hypotheses for ether which contradict those held for matter, as in the case of the so-called "vortex-atom" theory, which asserts the possibility of "frictionless" or unresisted motion.

The absolute view of objects as existing independently of the system of objects carries with it the belief that the motion of an object is similarly independent. Even in modern times it has seemed unnecessary to set up any indispensable relation between motion and resistance to motion, whence physicists are able to propound hypotheses regarding matter which accept or reject at will the notion that movement is possible in an existence vacuum. Nor does the ability of the mind to do this depend altogether on the survival of the old thought of the ether system as a vast emptiness whose only function is to allow itself to be occupied by objects. If the object be itself absolute—related only adventitiously and not constitutively to other objects—then its motion must in like manner be absolute, and we may conceive of it as being utterly indifferent to the presence or absence of resistance—may conceive of it, that

is to say, as motion against resistance, or as motion without resistance, according to the assumed circumstances. The important question whether the resistance is constitutively bound up with the motion as an essential and therefore indispensable element thereof, thus turns on the question whether the object which moves exists absolutely, or only by virtue of some relation which connects it with the system of objects. If, moreover, each unit object be regarded as possessing its nature as such absolutely, instead of contingently upon the system; and if motion be also viewed absolutely, as a character not constitutively related either to the nature of the object or to the nature of the system; then physicists are compelled to resort to one of two hypotheses—either to regard motion as an original, underived property of matter, possessed by each unit object, or to treat it as divinely originated at some period in the early history of the cosmos. The latter view is now widely rejected as anthropomorphic; as to the former, it will suffice to say that, besides being a negation of inertia, it is an effort to set up for unit objects in the "celestial spaces," characters which are denied by our experience of such objects on the surface of the earth. The law of likeness requires us to recognize the fundamental oneness of all the forms of power, yet physicists have thus far failed to unify the power which an object puts forth as moving with the power which the same object possesses and exerts by virtue of its nature as related to all other objects. Meanwhile the principle of derivation has been ignored in the effort to make all power kinetic by deducing it from motion instead of from substance.

IV.

This question of the relation of motion to the resistance which it encounters is always met with whenever we come to discuss such problems as the origin of motion, the direction of motion, the conservation of energy, inertia, and cause. As it is of the utmost importance for us to reach relational rather than absolute solutions of such problems, we shall lose nothing by re-emphasizing, in a special form for each, the constitutive relation whereby motion and resistance inter-condition and inter-involve each other. Note first

the difference between the absolute and the relative view of the origin of motion; in the absolute view motion is adventitiously acquired from the system, and is under no necessity of returning its energy to the system, whereas in the relational view the energy of motion, being from the system, must be returned to it. Our modern account, again, of motion as taking place along, or as following, the line of least resistance, cannot be regarded as other than overwhelmingly absolute. By distinguishing with such definiteness between the initial stress which results in motion and the resistance which the moving object encounters, it makes the so-called "resistance" adventitious merely—something which, so to speak, motion finds it has to deal with after it has once been set up. To say that motion follows the line of least resistance is to assume that the "least resistance" is something superadded to a process which already exists *de toutes pièces,* is therefore complete independently of whether there is resistance or none—in advance, that is to say, of any experience of resistance by the moving object. The absolute view regards motion as a character *sui generis*—as something which must happen whether it be "resisted" or not—as a change which, while it frequently takes place through a resisting medium, may also take place through an existence-vacuum—as something which is resisted not because "resistance" is any necessary, indispensable part of the motion process, but only because, under certain conditions, "resistance" happens to be present. The absolute view thus relates the "line of least resistance" adventitiously to the stress producing motion; the relational view relates it constitutively to that stress, and does this by showing that resistance, re-stress, or re-action is an essential condition of the stress which produces motion, and therefore an indispensable element of the motion process. The intellectual conditions of the problem, moreover, do not permit of an absolute view of motion, since were it possible for bodies to move through "empty space," the motion of such objects, being unresisted, would require no stress to initiate it, and we should be compelled to conceive of them as indifferently at rest or in motion—as lacking definiteness of condition.

An inevitable result of the view which regards the unit object

as possessing its properties absolutely, which disconnects the power of motion from the power of substance, and finds no constitutive necessity for a relation between matter and ether, is the absolute attitude of modern physics towards the method of the so-called "conservation of energy." The term "conservation" is itself objectionable, since it makes half the process do duty for the whole process. Not only must energy be expended in order to be conserved—there is no energy in any form save to the extent that "expenditure" and "conservation" take place simultaneously. Power, in a word, whether it be the power of objects or of the motion of objects, can exist only in the degree of a conserving process which involves expenditure, and of an expending process which involves conservation. To speak of a "conservation" of energy, moreover, is to suggest that, were the conditions unfavorable, energy might fail to be "conserved," might pass out of existence—in which supposition there is at once the error of implying that energy is complete before it is conserved, and the error of denying that presence of a conserving process which is the indispensable condition of expenditure. Note also that the absolute view provides two methods of conservation instead of one—methods wholly irreconcilable with each other —the distribution of the stress into the resistance, when there is resistance; and the continuance of the motion when there is "no resistance." It thus treats "expenditure" and "conservation" as separate processes either of which might theoretically exist without the other; it therefore regards the relation between them as adventitious. In the relational view they are constitutively related, since they reciprocally interinvolve each other as aspects of the two-sided unity which we call Power.

Observe now some of the results which the absolute attitude towards "conservation" makes inevitable. If "conservation of energy" is a character possessed absolutely, not contingently, by matter, then it is useless to seek the explanation of it in any process outside the realm of matter. If, again, the power with which an object moves is something *sui generis*—not related either to the nature of the object or to the power of substance—it is equally useless to seek for any relation between the "conservation" of the energy of

motion and the "conservation" of mass (the so-called indestructibility of matter). And if, finally, the conservation of the organism is in like manner something absolute, instead of contingent, there need be no inquiry as to its origin in earlier and more universal forms of conservation. It is, in fact, by failing to relate to each other the various forms of conservation—conservation of ether, conservation of motion, conservation of the organism—that science ignores the principles of likeness, derivation, and universality, and therefore abandons all hope of the solutions to which the application of these principles inevitably lead.

Here we reach the conception of inertia, which is similarly open to an absolute and a relational interpretation. Bodies are said to "persist" in their condition of rest or motion until that condition is changed by power other than their own—to go on moving in a straight line until they meet with resistance or are made to change their condition by force. All such propositions are characteristically absolute: they warn us of the possibility of resistance (reaction), but say nothing of its necessity. They assume that the moving object, with a convenient existence-vacuum for its "medium," may go on moving for ever; yet they fail to show how motion is possible to an object which the proposition has disconnected from the universe. In the absolute view, lingering everywhere in modern physics, the object is held to possess its condition of rest or of movement independently of its relation to the system, or at any rate, as a result of adventitious, rather than constitutive relation to the system; the continuous motion of an object is therefore attributed to absence of resistance, instead of to continuity of differential stress, while the continuous rest of an object is held to be due to absence of differential stress, instead of to continuity of equalized stress. All of which is opposed to the relational view, which holds that when an object remains at rest, it is held at rest by the system —that when it remains or continues in motion, it is kept in motion by the system. For inertia, instead of being a condition adventitiously acquired from the system, is a condition constitutively imposed by the system.

V.

The neglect to recognize constitutive relation is also seen in our modern attitude towards the organism, which has long been viewed absolutely—viewed rather as imposed upon nature, with powers possessed independently of nature, than as arising out of nature with such powers contingently originated. Hence the effort to explain the organism from the cell rather than the cell from the organism — an effort obviously founded on the belief, however tacitly held, that the cells are not constitutively, but only adventitiously related. This attitude is manifest in the perpetual striving of biological experts to find in some detail of cell structure the "secret of life"; in the belief, which confounds the concomitants of phenomena with their causes, that life is to be traced to some electrical, fermentive or other local process in the organism. We are here reminded of the slow advance which has been made towards relational views since the time when the organs of body and mind were treated by physicians as if each existed and possessed its powers more or less independently of the rest—when specialists expert in one field of human pathology were not expected to know anything of the other fields—when sociology, essentially a science of human relations, was not yet born, and it was the custom to regard the characters of the human being as individually intelligible, instead of as being explicable only in the light of the society of individuals. As early hypotheses regarding man left out of consideration the world of the lower organic life, in the supposition that man was self-sourced, instead of system-sourced, so theorists in the realm of human society took that absolute view of the individual human being out of which the exaggerated individualism of the eighteenth and nineteenth centuries had its birth. Even to-day our science of sociology still treats the human individual as related more or less adventitiously instead of constitutively to the sum of individuals, the biological idea, itself absolute, of a system of co-operating units, each complete in itself, having been incorporated almost bodily into the scientific conception of the social system.

Note also how the absolute view has retarded our knowledge of the process of organic development. In the theory of "natural selection" we have an explanation contrived for organisms which has no application to anorgana—a law which, being *ad rem,* describes no part of that higher, that more universal process in which organic development must have originated. And it is because of this isolation of the special theory from any more general theory that we have been compelled to ignore the problem of the manner in which variations arise in the attention which we give to the manner in which they are accumulated—to believe, in a word, that we can solve the problem of the "origin of species" without having first solved the problem of the origin of life. Instead of seeking in the world of the inorganic for some fundamental process of which the superficial organic process is merely a highly special and derivative form, we have been trying to explain the organism by dealing with it alone—by treating it as an object *sui generis,* as an object totally unrelated, or related only adventitiously, to the general system of objects which we call nature.

It is also in the very temper of the absolute view that we have dealt with the problem of intelligence. The principle of derivation requires us to explain the conscious from the unconscious forms of intelligence, yet we have never succeeded in ridding ourselves of the notion that some form of conscious intelligence, if not our own, is at work in the countless adaptations of structure and process which we find within the bodies of plants and animals. In like manner, the principle of derivation requires us to refer the general intelligence of the organism, conscious as well as unconscious, to that larger world in which it must have had its origin; yet instead of tracing organic intelligence to the inorganic cosmos, where the fundamental characters of that intelligence are plainly manifest, we try to read into the universe the most local, the most special, the most transitory forms of intelligent activity—those associated with the consciousness of the human organism. For scientific purposes, that is to say, we seek to explain the intelligence of the organism by making the atoms of matter intelligent; for general purposes, we try to understand nature by referring its intelligent

adaptations to a divine intelligence patterned after the intelligence of man.

VI.

The contrast thus shown between absolute and relational views may be traced, with simultaneous disclosure of the source of inaccuracy in the former, through every department of human knowledge. Everywhere the absolute method is the method of ignoring the dependence of the individual object, the single process, on the sum of processes, the system of objects; the relational view is everywhere the method of viewing objects and processes in the light of the totality of processes and objects. Advance in nature knowledge, however darkened and disguised by imperfect method, has ever been advance from absolute to relational ways of viewing nature; it has always meant progress from the thought of the object as self-sourced to the thought of it as system-sourced; it has never been anything other than the progressive mastery, by recognition of fundamental likeness, of those relations of stress and re-stress, of action and re-action, which make up the total system of power which we call universe. Nor is the contrast between absolute and relational merely a character of mental processes: it has its foundation and source in the world of objects. The whole of so-called development, alike in inorganic and organic, is nothing other than a gradual setting up of relation between units and groups—a growing dependence of each unit on the sum of units,—a gradual increase of the contribution made by unit to group, by group to unit—a progressive subordination of unit to system—an advancing domination of unit by system. The formation of matter groups by the gravitational and chemical processes is simply the earlier form of the process which manifests itself on a larger scale in the phenomena of crystallization, and on its most complex scale in the organism and in human society. In all the movements with which sociology deals—movements industrial, political, philanthropic, religious—life shows itself as a moving away from conditions that are absolute towards conditions that are relational. As all failures of nature knowledge have been failures of the absolute view, so all

failures in conduct, individual and national, have resulted from an inadequate sense of relation — from the effort to substitute for system-sourced conduct the conduct which is sourced in self. Movement away from the view of the individual and the group as self-dependent towards the view of both as contingent upon and dominated by the system—this is the process which has been at the heart of all moral advance.

It is now proposed to rationalize, and therefore to universalize the procedure which has thus far been followed in the investigation of nature—to make application to the cosmos as a whole the method which has hitherto been applied only locally and fragmentarily, and has thus been applied with inadequate realization of its universal meaning and scope. In the relational view, the solution of nature problems yet unsolved is nothing other than a process of reaching out from the individual object, the individual process, to the system which is to elucidate both—a passing from the local and superficial characters which most easily affect the senses to those fundamental and universal characters which appeal to the intellect—a process of universalization which, demanding fundamental likeness as the condition of its possibility, implies a universe of one kind throughout whose units, everywhere system-contributing and system-determined, exist only as they are constitutively interdependent.

Reasserting, therefore, the supremacy of the total mind process over the deliverances of the senses, as well as over every imperfect application of the intellect to those deliverances, the relational view demands that our knowledge of nature, instead of being made up of unrelated or imperfectly related views, shall have its foundation in an outlook over the whole field. Holding that the warrant for any view of nature must always be the reasonableness of that view—its consistency with all other attainable truths, with all other propositions believed to be true—it casts the light of the relational method over the path hitherto traversed by the mind. It indicates the way yet to be followed; it points also the goal to be reached. Its task is that of completing the monistic account of nature, and of doing this by revealing that cosmic unity which the accumulated facts of science, when relationally interpreted, already disclose.

The aim of the relational philosophy, so far as concrete nature problems are concerned, is to relate motion to the forms as well as to the substance of the cosmos; to correlate gravitation with light, electro-magnetism, heat; to connect chemical with electro-magnetic action; to unify the self-maintaining organism with the self-maintaining universe; to revise the theory of natural selection with such an account of the internal factors of organic development as shall separate the process by which intelligent adaptations are accumulated from the process by which they are originated; to universalize so-called "intelligence" by showing it to be primarily neither a conscious nor even an organic process, but a process rooted in the very nature of power; and finally, without danger of "pantheistic absorption" or injury to moral interests, to derive all organic and inorganic characters — characters of form, characters of motion, characters of mind—from their fountain and source, the substance system, the Creative Universe.

EDMUND NOBLE.

BOSTON, MASS.

ON THE FOUNDATIONS OF LOGIC AND ARITHMETIC.[1]

WHILE to-day in researches on the foundations of geometry we are essentially agreed as to the procedures to be adopted and the ends to be sought, it is quite otherwise with the inquiry concerning the foundations of arithmetic: here even yet the most diverse notions of the investigators stand bluntly opposed to each other.

The difficulties in the founding of arithmetic are partly indeed of a different character from those which were to be overcome in the founding of geometry.

In the examination of the foundations of geometry it was possible to leave aside certain difficulties of a purely arithmetical nature; in the founding of arithmetic, however, the appeal to another basal science seems unallowable.

I shall show the essential difficulties in the founding of arithmetic most clearly by subjecting to a brief critical discussion the views of individual investigators.

L. Kronecker, you know, saw in the concept of the whole number the true foundation of arithmetic; he formed the conception, that the integer, and that too as a general notion (parameter value), is directly and immediately given; thereby he was prevented from recognizing, that the idea of the whole number needs and is susceptible of a foundation. In so far I would designate him as a *dogmatist*: he takes the integer with its essential properties as dogma and makes no attempt to get behind it.

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[1] Translated by George Bruce Halsted.

H. Helmholtz represents the standpoint of the *empiricist;* the standpoint of pure experience, however, seems to me to be refuted by the objection, that from experience, that is, through experiment, can never be gotten the possibility or the existence of an indefinitely great number. For the number of the things which are object of our experience, even though it is great, lies nevertheless below a finite limit.

E. B. Christoffel and all those opponents of Kronecker's, who, led by the correct feeling, that without the concept of the irrational number the whole of analysis would remain doomed to unfruitfulness, seek, by finding out "positive" properties of this concept or through like means to save the existence of the irrational number, I would designate as *opportunists.*

In my opinion, however, they have not succeeded in reaching a real refutation of the Kronecker standpoint.

Among the scientists who have penetrated more deeply into the essence of the whole number, I may mention the following:

G. Frege sets himself the problem of founding the laws of arithmetic by means of *logic,* this taken in the usual sense. He has the merit of having rightly apprehended the essential properties of the concept of the whole number as well as the significance of the inference by complete induction. Inasmuch as he, however, true to his plan, takes this also among others as axiom, that a concept (an aggregate) is defined and immediately available, provided only it be determined for every object, whether it falls under the concept or not, and also in doing this subjects the concept "every" to no restriction, he exposes himself to just those paradoxes of the theory of aggregates, which lie, for instance, in the concept of the aggregate of all aggregates and which, it seems to me, show that the conceptions and means of investigation of logic, taken in the usual sense, are not adequate to the rigorous requirements set up by the theory of aggregates.

The avoidance of such contradictions and the clearing up of those paradoxes is rather from the very outset to be fixed upon as a chief aim in researches on the number concept.

R. Dedekind has clearly perceived the mathematical difficulties

in the founding of the number concept and in most ingenious fashion first supplied a construction of the theory of whole numbers.

I would, however, designate his as a *transcendental* method in so far as he conducts his proof for the existence of the infinite in a way, whose fundamental idea is indeed used in like manner by philosophers, but which because of the unavoidable contradiction of the concept therein employed of the totality of all things, I cannot acknowledge as allowable and sure.

G. Cantor has perceived the above-mentioned contradiction and has given expression to this perception by distinguishing between "consistent" and "inconsistent" aggregates. Inasmuch as he, however, in my opinion sets up no sharp criterion for this distinction, I must designate his conception on this point as one which still leaves room for the *subjective* judgment and therefore affords no objective certainty.

I am of the opinion all the difficulties touched upon can be overcome and we can attain to a rigorous and entirely satisfactory founding of the number concept, and that by a method, which I would call *axiomatic,* whose fundamental idea I wish briefly to develop in what follows.

Arithmetic is indeed designated as a part of logic and it is customary to presuppose in founding artithmetic the traditional fundamental principles of logic.

But on attentive consideration we become aware, that in the usual exposition of the laws of logic certain fundamental concepts of arithmetic are already employed, for example the concept of the aggregate, in part also the concept of number.

We fall thus into a vicious circle and therefore to avoid paradoxes a partly simultaneous development of the laws of logic and arithmetic is requisite.

In the brief space of a lecture I can merely indicate how I conceive of this common construction. Therefore I ask indulgence if I succeed only in giving you a rough notion of what direction my researches are taking. Moreover, for the sake of being more easily understood, I shall employ the ordinary speech "in words"

and the laws of logic therein indirectly expressed, more than would be desirable in an exact construction.

Let an object of our thinking be called a *thought-thing* or briefly a *thing* and designated by a symbol.

Let us take as the basis of our consideration first of all a thought-thing I (one).

The taking of this thing together with itself respectively two, three or more times, as:

II, III, IIII,

we designate as *combinations* of the thing I with itself; in like manner any combinations of these combinations, as:

(I)(II), (II)(II)(II), ((II)(II))(II), ((III)(I))(I)

are in turn called combinations of this thing I with itself.

The combinations likewise are designated merely as things and then in distinction to this the fundamental thought-thing I as *simple* thing.

We adjoin now a second simple thought-thing and denote it by the symbol =(equal). We consider now in turn the combinations of these two thought-things, as:

I=, II=, ...(I)(=I)(===), ((II)(I)(=))(==), I=I,
(II)=(I)(I).

We say, the combination a of the simple things I, = *differs* from the combination b of those things, if they, as regards the mode and sequence of the combination, or the choice and participation of the things I, = themselves, deviate in any way from one another, that is if a and b are not *identical* with each other.

Now let us think the things I, = and their combinations as somehow divided into two classes, *the class of the existent* and *the class of the non-existent*: everything which belongs to the class of the existent, differs from everything which belongs to the class of the non-existent. Every combination of the two simple things I, = belongs to one of these two classes.

If a is a combination of the two fundamental things I, =, then we designate also by a the *statement*, that a belongs to the class

of the existent, and by $\bar{a}$ the *statement,* that a belongs to the class of the non-existent. We designate a as a *true* statement, if a belongs to the class of the existent; on the other hand let $\bar{a}$ be called a *true* statement, if a belongs to the class of the non-existent.

The statements a and $\bar{a}$ constitute a *contradiction.*

The composite from two statements, A, B, represented in symbols by

$$A|B,$$

in words: "from A follows B" or "if A is true, B also is true" is likewise called a statement and then A is called the *hypothesis,* B the *conclusion.*

Hypothesis and conclusion may themselves in turn consist of several statements A_1, A_2, respectively B_1, B_2, B_3 and so forth, in symbols:

$$A_1 \,\&\, A_2 \mid B_1 \text{ o. } B_2 \text{ o. } B_3,$$

in words: "from A_1 and A_2 follows B_1, or B_2, or B_3" and so forth.

In consequence of the symbol o. (or) it would be possible, since negation is already introduced, to avoid the symbol |; I use it in this lecture merely to follow as much as possible the customary word-speech.

We will understand by A_1, A_2,... respectively those statements which—to be brief—result from a statement $A(x)$ when in place of the "arbitrary" x we take the thought-things 1, $=$ and their combinations; then we write the statements

$$A_1 \text{ o. } A_2 \text{ o. } A_3,\ldots \text{ respectively } A_1 \,\&\, A_2 \,\&\, A_3,\ldots$$

also, as follows:

$A(x^{(o)})$, in words "at least for one x"
respectively $A(x^{(\&)})$, in words "for every single x,"

in this we see merely an abbreviated way of writing.

We make now from the fundamental two things 1, $=$ the following statements:

1. $x = x$
2. $[x = y \,\&\, w\,(x)] \mid w(y).$

Therein x (in the sense of $x^{(\&)}$) means each of the two funda-

mental thought-things and every combination of them; in 2. y (in the sense of $y^{(\&)}$) is likewise each of those things and each combination, furthermore $w(x)$ an "arbitrary" combination, which contains the "arbitrary" x, (in the sense of $x^{(\&)}$); the statement 2. reads in words:

From $x = y$ and $w(x)$ follows $w(y)$.

The statements 1., 2. form the *definition of the concept* = (equal) and are in so far also called *axioms.*

If one puts in place of the arbitraries x, y in the axioms 1., 2. the simple things 1, = or particular combinations of them, there result particular statements, which may be called *inferences* from those axioms.

We consider a series of certain inferences of such a sort, that the hypotheses of the last inference of the series are identical with the conclusions of the preceding inferences.

Then if we take the hypotheses of the preceding inferences as hypothesis and the conclusion of the last inference as conclusion, there results a new statement, which in turn may be designated as an inference from the axioms.

By continuation of this deduction-process we may obtain further inferences.

We select now from these inferences those which have the simple form of the statement a (affirmation without hypothesis), and comprehend the things a so resulting in the class of the existent, while the things differing from these may belong to the class of the non-existent.

We recognize, that from 1., 2. only inferences of the form $\mathfrak{a} = \mathfrak{a}$ ever arise, where $\mathfrak{a}$ is a combination of the things 1, =.

The axioms 1., 2. in their turn as regards the partition in question of the things into the two classes are also fulfilled, that is true statements, and because of this property of the axioms 1., 2. we designate the concept = (equal) defined by them as a concept *free from contradiction.*

I would call attention to the fact, that the axioms 1., 2. do not at all contain a statement of the form $\bar{a}$, that is a statement, accord-

ing to which a combination is to be found in the class of the non-existent.

We therefore could also satisfy the axioms by comprehending the combinations of the two simple things all in the class of the existent and leaving the class of the non-existent empty.

The partition above chosen into the two classes, however, shows better how to proceed in the subsequent more difficult cases.

We now carry the construction of the logical foundations of mathematical thinking further, by adjoining to the two thought-things u (infinite aggregate, infinite), f (following), f′ (accompanying operation) and laying down for them the following axioms:

3. $f(ux) = u(f'x)$
4. $f(ux) = f(uy) \mid ux = uy$
5. $\overline{f(ux) = u1}$

Therein the arbitrary x (in the sense of $x^{(\&)}$) means each of the five now fundamental thought-things and every combination of them.

The thought-thing u may be called briefly *infinite aggregate* and the combination ux (for example u1, u(11), uf) an *element* of this infinite aggregate u.

The axiom 3. then expresses, that after each element ux follows a definite thought-thing $f(ux)$, which is equal to an element of the aggregate u, namely to the element $u(f'x)$, that is belongs likewise to the aggregate u.

The axiom 4. expresses the fact, that, if the same element follows two elements of the aggregate u, those elements also are equal.

According to axiom 5. there is no element in u, after which the element u1 follows; this element u1 may therefore be called the first element in u.

We have now to subject the axioms 1.—5. to the investigation corresponding to that before made of the axioms 1., 2.; therein it is to be noticed, that those axioms 1., 2. at the same time experience an extension of their validity, inasmuch as now the arbitraries x, y mean any combinations you please of the five fundamental simple things.

We ask again, whether certain inferences from the axioms 1.—5. make a contradiction or whether on the contrary the fundamental five thought-things 1, =, u, f, f′ and their combinations can be so distributed into the class of the existent and the class of the non-existent, that the axioms 1.—5. in regard to this partition into classes are fulfilled, that is, as regards that partition into classes, each inference from those axioms comes to be a true statement.

To answer this question, we take into account that axiom 5. is the only one which gives rise to statements of the form $\bar{a}$, that is that a combination a of the five fundamental thought-things is to belong to the class of the non-existent. Statements, which with 5. make a contradiction, must therefore in any case be of the form:

$$6.\quad f(ux^{(0)})=u1:$$

such an inference, however, can in no wise result from the axioms 1.—4.

In order to perceive this, we designate the equation, that is the thought-thing $a=b$ as a homogeneous equation when a and b are both combinations of two simple things, and also if a and b are both any combinations of three or both any combinations of four or more simple things; for example

$$(11)=(fu),\ (ff)=(uf'),\ (f11)=(u1=),\ (f1)(f1)=(1111),$$
$$[f(ff'u)]=(1uu1),\ [(ff)(111)]=[(1)(11)(11)],\ (fu111=)$$
$$=(uu111u)$$

are called homogeneous equations.

From the axioms 1. and 2. alone follow, as we have already seen, nothing but homogeneous equations, namely the equations of the form $a=a$. Just so axiom 3. gives only homogeneous equations if in it we take for x any one thought-thing.

Likewise axiom 4. is certain to exhibit in the conclusion always a homogeneous equation, if only the hypothesis is a homogeneous equation, and consequently only homogeneous equations can appear at all as inferences from the axioms 1.—4.

Now, however, the equation 6., which is the one to be proven, is certainly no homogeneous equation, since therein in place of $x^{(0)}$ one has to take a combination and thus the left side comes to be a

combination of three or more simple things, while the right side remains a combination of the two simple things u and 1.

Herewith is explained, as I think, the thought fundamental for the recognition of the correctness of my assertion; for the complete carrying through of the proof there is need of the idea of the finite ordinal number and certain theorems about the concept of equality as to number, which in fact at this point can without difficulty be set up and deduced: for the complete carrying through of the stated fundamental thought we have still to consider those points of view, to which I will briefly refer at the close of my lecture. (Compare V.)

The desired partition into classes results therefore, if one reckons as in the class of the existent all things a, where a is an inference from the axioms 1.—4., and considers as in the class of the non-existent all those things which differ from these, especially the things $f(ux)=u1$.

Because of the property of the assumed axioms so found, we recognize, that these never lead at all to a contradiction, and therefore we designate the thought-things u, f, f′ defined by them as concepts or operations *free from contradiction* or as *existing free from contradiction* (compatible).

As to the concept of the infinite u in particular, the affirmation of the *existence of the infinite* u thus appears justified through the above indicated exposition; for it gets now a definite meaning and a content continually to be applied later on.

The investigation just sketched makes the first case in which the direct proof of the freedom-from-contradiction of axioms has been successfully given, whereas the method heretofore usual—especially in geometry—for such proofs, that of suitable specialization or construction of examples, here necessarily fails.

That this direct proof here succeeds, is, as one sees, essentially owing to the circumstance, that a statement of the form $\bar{a}$, that is a statement, according to which a certain combination is to belong to the class of the non-existent, only appears as a conclusion in one place, namely in axiom 5.

When we translate the known axioms for complete induction into the speech chosen by me, we attain in like manner to the com-

patibility of the so increased axioms, that is to the proof of the contradiction-free *existence of the so-called smallest infinite** (that is, *of the ordinal type* 1, 2, 3, ...).

There is no difficulty in founding the concept of the finite ordinal number in accordance with the principles above set up; this is done on the basis of the axiom, that every aggregate which contains the first element of ordinal number and, in case any element belongs to it, also contains the one following this, surely must always contain the last element.

The proof of the compatibility of the axioms follows here very easily by the bringing in of an example, for instance of the number two. It is then the main point, to show, that an arrangement of the elements of the finite ordinal number is possible, such that each part-aggregate of it possesses a first and a last element—a fact, which we prove by defining a thought-thing $<$ by the axiom

$$(x < y \;\&\; y < z) \mid x < z$$

and then recognizing the compatibility of the axioms set up with the addition of this new axiom, when x, y, z mean arbitrary elements of the finite ordinal number.

By using the fact of the existence of the smallest infinite, the theorem then follows also, that for each finite ordinal number a still greater ordinal number can be found.

The principles which must be normative for the construction and further elaboration of the laws of mathematical thinking in the contemplated way, are briefly the following:

I. Arrived at a definite standpoint in the development of the theory, I may designate a further statement as true, as soon as is recognized, that it superadded as axiom to the statements already found true, gives no contradiction, that is leads to inferences, which in regard to a certain partition of things into the class of the existent and that of the non-existent are all true statements.

II. In the axioms the arbitraries—as equivalent for the concept "every" or "all" in the customary logic—represent only those

* Compare my lecture delivered before the International Congress of Mathematicians at Paris in 1900: *Mathematical Problems*, 2. The Compatibility of the Arithmetical Axioms.

thought-things and their combinations with one another, which at that stage are laid down as fundamental or are to be newly defined. Therefore in the deduction of inferences from the axioms, the arbitraries, which occur in the axioms, can be replaced only by such thought-things and their combinations.

Also we must duly remember, that through the superaddition and making fundamental of a new thought-thing the preceding axioms undergo an enlargement of their validity, and where necessary, are to be subjected to a change in conformity with the sense.

III. The aggregate is generally defined as a thought-thing m, and the combinations mx are called elements of the aggregate m, so that therefore—in opposition to the usual conception—the concept of the element of an aggregate appears only as later product of the idea of aggregate.

Exactly as the concept "aggregate" are also "correlation," "transformation," "reference," "function" thought-things for which, precisely as was done a moment ago with the concept "infinite," the suitable axioms are to be stated, and these then in the case of the possibility of the partition of the respective combinations into the class of the existent and that of the non-existent can be recognized as compatibly existing.

In I. the creative principle receives expression which in the freest application warrants us in ever new concept-building with the sole restriction of the avoidance of a contradiction. The paradoxes mentioned at the beginning of this lecture become because of II. and III. impossible; especially does this hold of the paradox of the aggregate of all aggregates not containing themselves as element.

In order to permit the perception of the far-going agreement in content of the concept of aggregate defined in III. with the usual aggregate-concept, I prove the following theorem:

At a definite stage in the development let

$$\mathfrak{1}, \ldots, \mathfrak{a}, \ldots, \mathfrak{k}$$

be the fundamental thought-things and $a(\xi)$ a combination of these, which contains the arbitrary ξ; further let $a(\mathfrak{a})$ be a true statement

(that is $a(a)$ in the class of the existent): then there is sure to exist a thought-thing m of such a sort, that $a(mx)$ for the arbitrary x represents true statements only (that is $a(mx)$ always occurs in the class of the existent) and also inversely each thing ξ, for which $a(\xi)$ represents a true statement, is equal to a combination $mx^{(o)}$, so that the statement

$$\xi = mx^{(o)}$$

is true, that is the things ξ, for which $a(\xi)$ is a true statement, make the elements of an aggregate m in the sense of the above definition.

In proof we set up the following axiom: m is a thought-thing, for which the statements

7. $a(\xi) \mid m\xi = \xi$
8. $\bar{a}(\xi) \mid m\xi = a$

are true, that is if ξ is a thing such that $a(\xi)$ belongs to the class of the existent, then must $m\xi = \xi$ hold good, otherwise $m\xi = a$; adjoin this axiom to the axioms which are valid for the things

$$1, \ldots, a, \ldots, k,$$

and then assume, that thereby a contradiction is produced, that is, that for the things

$$1, \ldots, a, \ldots, k, m$$

perchance the statements

$$p(m) \text{ and } \overline{p(m)}$$

are at one and the same time inferences, where $p(m)$ is a certain combination of the things

$$1, \ldots, k, m.$$

Therewith 8. means in words the stipulation $m\xi = a$, if $a(\xi)$ belongs to the class of the non-existent.

Whenever in $p(m)$ the thing m appears in the combination $m\xi$, replace in accordance with the axioms 7. and 8. and taking 2. into consideration the combination $m\xi$ by ξ, respectively a; if from $p(m)$ is formed in this way $q(m)$ (where now $q(m)$ no longer contains the thing m in a combination mx), then must $q(m)$ be an inference from the original fundamental axioms for

$$1, \ldots, a, \ldots, k$$

and therewith also remain true if we for m take any one of these things, as for instance the thing 1.

Since the same consideration holds also for the statement $\overline{p(m)}$, there would therefore exist also at the original stage, when we take as a basis the things

$$1, \ldots, a, \ldots, k,$$

the contradiction

$$q(1) \text{ and } \overline{q(1)},$$

which cannot be—it being presupposed that the things

$$1, \ldots . k$$

exist free from contradiction. We must therefore reject our assumption, that a contradiction is produced; in other words, m exists free from contradiction which was to be proved.

IV. If we wish to investigate a definitely given system of axioms in accordance with the above principles, then we must partition the combinations of the fundamental things into the two classes, that of the existent and that of the non-existent, and in this process the axioms play the rôle of prescriptions which the partition must satisfy.

The chief difficulty will consist in making out the possibility of the partition of all things into the two classes, that of the existent and that of the non-existent.

The question of the possibility of this partition is essentially equivalent to the question, whether the inferences, which can be obtained from the axioms through specialization and combination in the previously exemplified sense, lead to a contradiction or not, *if besides are adjoined the familiar logical deduction-modes such as*

$$[(a|b) \;\&\; (\bar{a}|b)]|b$$
$$[(a \text{ o. } b) \;\&\; (a \text{ o. } c)]|[a \text{ o. } (b \;\&\; c)].$$

The compatibility of the axioms can then either be made out by showing how a peradventure contradiction must show itself as early as a preceding stage in the development of the theory, or by making the assumption, that there is a proof, which leads from the axioms to a definite contradiction, and then demonstrating, that

such a proof is not possible, that is to say contains in itself a contradiction. Thus indeed the proof just now sketched for the contradiction-free existence of the infinite amounts also to making out, that a proof for the equation 6. from the axioms 1.—4. is not possible.

V. Whenever in what precedes *several* thought-things, combinations, combinations of *manifold* sort or *several* arbitraries were spoken of, a limited number of such things ought always to be understood.

After the setting up of the definition of the finite number we are in position to take that mode of expression in its general meaning.

Also the meaning of the "any you please" inference and of the "differing" of one statement from all statements of a certain kind is now, on the basis of the definition of the finite number—corresponding to the idea of complete induction—through a recurrent procedure, capable of an exact description.

Thus also is to be conceived the complete carrying through of the proof just now indicated, that the statement

$$f(ux^{(o)})=u1$$

differs from each statement which results through a finite number of steps as inference from the axioms 1.—4.: one has, that is, to consider the proof itself as a mathematical structure, namely a finite aggregate, whose elements are connected through statements expressing that the proof leads from 1.—4. to 6., and one has then to show that such a proof contains a contradiction and therefore does not in our defined sense exist free from contradiction.

In a way like that in which the existence of the smallest infinite can be proven, follows the existence of the assemblage of real numbers: in fact the axioms as I have set them up* for real numbers are expressible precisely through such formulas as the axioms hitherto laid down. As for that axiom which I have called the axiom of completeness, it expresses that the assemblage of real

* *Grundlagen der Geometrie*, second edition, Leipsic, 1903, pp. 24-26.

numbers in the sense of the reversible unique referability by elements contains every other aggregate whose elements likewise fulfill the preceding axioms; thus conceived the axiom of completeness also becomes a requirement expressible through formulas of the foregoing structure and the axioms for the assemblage of real numbers are qualitatively distinguished in no respect from the axioms necessary for the definition of the whole numbers.

In the perception of this fact lies, as I think, the real refutation of the conception of the foundations of arithmetic advocated by L. Kronecker and at the beginning of my lecture designated as dogmatic.

In like manner is shown, that contradiction-free existence belongs to the fundamental concepts of the Cantor theory of aggregates, in particular to the Cantor alefs.

D. HILBERT.

GÖTTINGEN.

SHYLOCK IN THE OLD TESTAMENT.

THE STORY OF LABAN VIEWED IN THE LIGHT OF THE CODE HAMMURABI.

A VERY interesting question concerning the relations of the famous Hammurabi Code to the Hebrew record has hitherto been passed over by critical students. The intrinsic interest of the narrative, when read in the light of the law of the time in that region, as well as its value as material for critical purposes, suggest that it should be given careful consideration. For the present paper, a bare presentation of parallels must answer: critical conclusions being left to the reader.

The Hebrew traditions declare that their ancestors were Arameans. "An Aramean ready to perish was my father," was the confession of the devout Hebrew. "Laban the Aramean" is of Abraham's own kindred. The point of departure, geographically speaking, is Ur-Casdim. The various theories about the location of Ur need not be discussed here. The preponderance of opinion is toward a location in southern Babylonia. We are therefore justified in examining the early Aramean stage of the Hebrew people for traces of Babylonian influences. Conversely, should comparison of the Code of Hammurabi with the patriarchal narratives disclose in the latter evidences of Babylonian culture, we would be strengthened in the view that the site of Ur-Casdim must be sought in Babylonia.

The theory that a primitive common clan law sufficiently accounts for all resemblances between the Babylonian and the patriarchal law can also be tested. If they are but sister developments

from a common nomad code, we should not anticipate wide differences in patriarchal practice in the same generation. But if kinsmen dwelling in different lands are shown to speak and act as though the laws of their respective homes were widely different, we may be sure that we have to deal with considerably developed local laws, instead of mere primitive clan customs.

Hammurabi's own records also warrant our making the proposed examination. "Martu," as Palestine was called, is mentioned repeatedly in his inscriptions. The precise extent of Babylonian authority there we do not know. But Harran and Aleppo are mentioned by Hammurabi as objects of his special care and attention. We find that Babylonian supremacy seems complete throughout the Euphrates valley: and we may expect all important transactions in that region to be influenced by the Hammurabi Code, while a stranger from Southern Palestine would probably not be familiar with the law of the land. Only one of the patriarchal narratives, that of Jacob and Laban, has its scene laid in the land of Harran. The sharp practice narrated should then be examined in the light of Harran law.

In the Code of Hammurabi, much attention is given to the property rights of women. We are concerned with but one of the elements of a woman's property in the present case, the *tirhatu,* or betrothal present. The groom is expected to bring such at the time of the formal arrangement of the marriage contracts. The father of the bride must not appropriate this gift, and must endow his daughter's marriage, else he puts her in the position of having been sold as a slave, instead of honorably married.

The bride's possession of the *tirhatu* is clearly indicated by the general principle of the code, which compels a wrong-doer to forfeit to the injured party a sum at least equal to the amount originally involved in any given business transaction. In the cases where a wife is wronged by her husband, the equivalent of the original *tirhatu* is paid over to the wife herself, not to her father. If the latter holds the *tirhatu,* it seems that he does so merely in trust; to divert it to his own uses would be equivalent to embezzlement. If the wife be divorced without just cause, the *tirhatu* is doubled

by the husband and paid over to the wife as penalty for infringement of the marriage contract.

One naturally asks, with regard to the *tirhatu,* what were the poor man's chances for marrying into a wealthy family, or what he would do if unable to purchase a maid-servant. The code makes it apparent that some equivalent might be offered by a penniless suitor. In section 139 we are told, in case of unjust divorce, that if no *tirhatu* had been presented by the husband in the beginning, one mina of silver must be paid to the divorced wife. That is to say, one mina of silver is named as the equivalent of the least *tirhatu* which should have been brought by the penniless suitor.

Now from the wage-scale in the code we learn that the standard daily wage for a first-class herdsman or agriculturist is 5 ŠE of silver per day; skilled artisans being rated slightly lower. As 180 ŠE make one shekel, and 60 shekels a mina, and, as Prof. Morris Jastrow has shown (A. J. T., 1898), that the Babylonians observed a seventh day in some fashion, we readily recognize that one shekel means six weeks wages and one mina 360 weeks; or exactly seven years, at the ordinary Semitic standard of 360 days to the year. Seven years service, for a day-laborer who is a competent workman, stands then as the possible alternative for a *tirhatu* in cash or goods, if his aspirations are to alliance with one in more fortunate circumstances. But what has been pointed out with regard to the bride's ownership of the *tirhatu* would indicate that the net proceeds of such service the bride would consider as constituting her *tirhatu;* and appropriation of it by her father she would be prepared to resent.

In the story of Jacob and Laban, the former, after a month's residence, proffered seven years service for the hand of Rachel. Perhaps Laban felt that the minimum amount proffered was derogatory to the dignity of his family. Whatever the motives of his conduct, it appears that Jacob did not know the laws of the land, and Laban was prepared to take advantage of the fact. He had the law constructively in favor of his specious excuse. While not specifically enjoined, it is clearly expected (C. H. 66) that the marriages of the older children shall be arranged first.

Laban has the further advantage in the fact that no bonds appear to have been drawn up, specifying which daughter Jacob desired as his bride. C. H. 128 declares that if a man has betrothed or married a wife, but has not drawn up the required bonds, that woman is no wife. In the absence of such documents Jacob could not have a legal claim to Rachel.

Jacob's reliance upon a mere verbal agreement between kinsmen in a land whose laws made witnesses or bonds essential to the validity of all important business transactions (C. H., *passim*) had brought him a bitter experience. By the end of the second seven years, however, it is apparent that he knew something of the technicalities of the law himself, and was prepared to better the instruction by putting Laban in the position of breaking the law repeatedly. Two important sections of the code may be quoted here (C. H. 264, 265):

"If a herdsman, to whom oxen and sheep have been given to pasture, receives his hire according to agreement, is satisfied (in that particular), and allows the cattle or sheep to become enfeebled, or lessens the birth-rate, according to his contracts he shall make good the birth-rate and the increase."

"If a herdsman, to whom oxen or sheep have been given to pasture, has been dishonest, or has altered the terms (of his contract) or has sold them, they shall call him to account, and he shall restore to the owner oxen or sheep tenfold what he had stolen."

These provisions are not intended for the ordinary day laborer, but for the master-herdsman or contractor. Laban's experience with Jacob had convinced him that he had a valuable master-herdsman whom it might be well to keep at his own terms. The sections quoted indicate the customary character of such agreements. It is expected that contracts shall stipulate a certain reasonable proportion or percentage of increase. With such a contract, Jacob would have no opportunity for his revenge. Hence the peculiar proposition he makes to Laban.

Now if Jacob proposes to alter the terms of his contract, he is liable for heavy damages; but he so manipulates matters that Laban changes the terms himself, ten times over; and Jacob has Laban's

daughters as witnesses to the fact (Gen. xxxi. 6, 7). Again, if Jacob, under an ordinary contract, diminishes or enfeebles the cattle, Laban can collect damages from him; but the character of the agreement leaves Laban no such recourse, and he repeatedly alters the terms to recover his previous losses.

Meanwhile Laban's daughters have a deep sense of wrong. No dowry had been given them: the results of Jacob's service had been appropriated by Laban; and they were in consequence in the status of maid-servants, who had been sold to a foreigner, contrary to the provisions of the code (C. H., 280). Or they might consider themselves as legally in the status of unportioned concubines whose marriage their father refused to endow, and who must expect their brothers, after their father's death, to give them their rights (C. H., 184). But the black looks of their brothers, who are asserting that Jacob has robbed Laban, warn them that they need not look for redress in that quarter. At the family council the wives of Jacob apparently approve his methods, declaring that their father has treated them as strangers or foreigners, that he has devoured or wholly alienated all that they rightfully considered their own, and that he has simply sold his daughters as if they were slaves. Jacob, in their view, is only helping himself in a case where the law could not help him, when he plans to get possession of the vast increase in Laban's cattle, which his service had produced, as their *tirhatu*.

But there is another contingency, which they apparently consider: Laban's original fraud, with the absence of legal documents, makes it possible for Laban to take radical action in the case. He might choose, in the last resort, to take away his daughters by force, upon the ground that they were not legally married, and so send Jacob away stripped of everything. As the strained relations make it clear that the end is nigh, Rachel, the ancient Jessica, cunning and daring as well as beautiful, plans the final *coup*.

This may be understood from C. H. 6. We learn that whoever steals the property or household furnishings of a god, shall be put to death, and whoever receives the stolen goods shall be put to death. Such property of the god would include not merely symbols and statuettes, but all kinds of votive or dedicated objects.

Babylonian Judaism in a later period shows how wide a range of interpretation could be given the law. It could be made to accord to property as well as to person the privilege of sanctuary. Unprincipled Jews gave it great abuse. Christ alludes to it in his reference to "Corban," "dedicated," being made an excuse by unnatural children for not using their property to help a needy parent. Perhaps this originated the other device of a "gift to Cæsar" to forestall confiscation by some greedy official. Such protection of valuables may have been the end sought by Micah's wealthy mother, in the story in Judges. Learning the fact, and of the terrible curses launched against the thief, the frightened Micah hastened to surrender his plunder.

The reader may imagine the frame of mind of Laban when he learned that Jacob and Rachel had outwitted him, and had a week the start in the race for the frontier. One may apply, if he will, the outcry of Shylock as described by Salanio. But in the present case —perhaps because Jacob has the stronger troop, and because efforts to recover any property without violence would lead to recriminations that would not be helpful to his cause,—Laban discovers that he has religious scruples upon certain points, and will speak neither good nor bad. His insincerity is shown by his boasting of power to do hurt to the fugitives, and by his immediate preferment of a capital charge.

Jacob replies, stating that he had feared forcible action upon the part of Laban, and later tells him openly that Laban would, if he dared, strip him of everything. Of the final offense charged by Laban he knows nothing, but admits its capital character: "With-whomsoever thou findest thy gods, let him not live." Legal action concerning all earlier proceedings Jacob does not fear, as the absence of documents in the original fraud destroys Laban's opportunities for appeal to the law. Laban cannot deny the bringing of a *tirhatu* in the form of seven years' service. If he choose to assert that the marriage is not legally complete, and that he prefers to bestow his daughter upon some one else, the law would compel him to refund the *tirhatu* or proceeds of Jacob's service, and to add to

it an equal sum (C. H., 160, 161). Such course would be out of the question for such as Laban.

To grasp at once the sudden change of the *rôles* of prosecutor and prosecuted in the dramatic *dénouement*, it must be remembered that the *lex talionis* is the fundamental principle of the code, and every formally preferred charge, if unproven, recoils with the corresponding penalty upon the head of the accuser. C. H. 1 declares that if a man charge a capital crime upon any one and fail to prove it he shall be put to death.

At the end of the search, Jacob suddenly turns upon his accuser, and in his lofty rage he assumes the *rôle* of the magnanimous, as he demands instant trial, before witnesses. Laban has charged a capital crime, and boasted of his power to hurt before the whole company. His charge is not proven and his life is legally forfeit. The herdsman's contracts between them have been altered repeatedly, despite the most faithful service upon Jacob's part; each alteration is liable to a fine of tenfold the amount involved. Laban's sons have been charging fraud (Gen. xxxi. 1); they should prove their allegations, or the family may be fined a sum equal to that which they assert lost. The aggregate fines for the charges made would bankrupt the family. Jacob goes further. The law does not hold the shepherd responsible if wild beasts slay his employer's cattle (C. H., 266). Yet Jacob declares that he voluntarily assumed such losses at first, and that Laban later required it, contrary to the law. He then reverts to the original fraud and closes by asserting that Laban would, if he dared, rob him of everything. Thus the avaricious Aramean, very punctilious about small technicalities of the law when he would get the advantage of his foreign kinsman who did not know the law of the land, has, without his suspecting it, been put in the position of breaking the law repeatedly. He is legally bankrupt, his life is forfeit; his jewelry and sacred equipment have been stolen by his fleeing daughter; and, despoiled of all opportunity for legal redress, he is compelled to sue for peace, and to ask for an agreement that there shall be no effort at prosecution.

In this request for a treaty we may observe another point of contact with the code. Under its provisions a man could not marry

two wives unless the first were a chronic invalid. But for Leah's weak eyes, (perhaps serious abscesses, a common malady of the eye in the Orient and especially considered in the code, 215-220) the first fraud of Laban could have permanently closed the way to Rachel. Neither as wife, maid or concubine could she have come to Jacob, under the law. Perhaps Laban had never intended that she should. But the advisability of retaining a skilful master herdsman would be enough to impel a man like Laban to avail himself of the very convenient section 148 of the code, using Leah's bad eyes as a pretext. But at the witness cairn of Gilead the frontier is near. Beyond the border the code does not apply, and other social standards prevail. That his daughters may not be mistreated, and that Jacob shall take no other wives, Laban has recourse to adjuration.

We may have here one version—perhaps the original—of that much varied, far traveled popular tale, which the genius of Shakespeare has immortalized for the English-speaking world in the "Merchant of Venice." The main elements—the contest of wits over legal technicalities between an avaricious old man and a young man of another land, who finally bests the elder man and runs away with the daughter and the family jewelry, with the dramatic *bouleversement* at the trial scene—are all in the patriarchal tale. It should prove a very interesting problem for the student of comparative folklore.

A. H. GODBEY.

CHICAGO.

QUALITY AND QUANTITY.[1]

THE problem of quality and quantity implies two distinct studies, which we shall combine here for greater convenience in exposition. The first is of an historical nature and must answer the question: What is, in the history of philosophy, the exact meaning of the opposition of the concepts of quality and quantity? or, in other words, what is the original attitude of modern physics toward the old conception? The second study, of a purely rational character, must solve the question, What is the psychological meaning of the words quality and quantity, and from a scientific point of view what is the import of the contrast of two categories so long admitted to be mutually irreducible? in other words, what must be thought of the transition from the unextended to the extended? or, in any case how is it brought about, and what is really the value of the distinction between intensive and extensive magnitude—the latter being clear and serving as an adequate symbol to the former which is as confused as the very depths of being?

I.

Quality and quantity characterize very exactly the two points of view, from which the human mind has successively regarded physics during the long history of the progress of that science taken in its most general sense. In turn each term becomes the crucial test as to the nature of matter: quality is the pivot around which the old science of physics gravitates; quantity, the new. Between these two conceptions has evolved the theory of living beings as well as that of inorganic bodies. It is enough to say that the

[1] Translated by Amélie Séraphon.

problem of the relation of quantity to quality is not lacking in interest. We will consider here only very general ideas.

These then are the terms of the psychological problem:

1. From the subjective point of view, trying to find quantity in quality will be looking for the application of the law of number or measure in the realm of mere intensity; it will be reducing to well-known proportions the mobile and intangible depths of sensation; it will be finding out the connection between the object exciting sensation, and the sensation itself. From this point of view, however, one particular will always escape us, and that is the purely emotional and affective in sensation.

2. From the objective point of view, searching for quantity in quality, will be passing from the subject to the object, by reducing to distinct and clear formulas the sensations that we interpret as signs of some object outside of ourselves; it will be affirming that we know objects only in relation to the clear ideas we have of them, and that the reality of the external world is connected with the evidence of the mathematical formulas to which we reduce them. This is again transferring the unextended into the extended, and making general mathematics the condition of all certitude. But from this point of view also, one thing will always escape us, namely: to know the very fundamental facts set in equation, and the necessity of this interdependence postulated *a priori* of reason and reality. Therein lies the great problem of thought.

Sound, color, odor, taste, cold and heat, resistance, weight, hardness, shape, situation, and movement: in these consists all our external world, or at least as much of it as can be perceived by a consciousness in which reflection has had no part. The distinction between the qualitative and the quantitative point of view that we establish in things, is not so clear on the whole as it would seem at first. Common sense, far from seeing a great difference between the extensive and intensive, makes of them two forms of one and the same thing, as it were. It immediately looks upon quantity and quality as notions fusing into the unity of an object of which they express different modes of existence on a common basis. Thus quality and quantity imply each other, since all ideas relating to

magnitude indicate as many ways of existing, and everything that serves to determine the nature of a thing is rightly called quality. Such is the common opinion. We shall see later, on what preconceived idea it rests. Now we shall limit ourselves to stating that this radical difference which is primarily established between quality and quantity is not founded on reason. What we first know are the sensations which we distinguish from what we suppose to be their causes; that is to say, we first perceive ourselves and afterwards that which is not ourselves.

II.

Before going further, let us examine what may fitly be called the distinction between primary and secondary qualities. Does such a distinction impress itself upon the popular mind? or is it but the sublimated result of reflecting thought?

It seems to me that common sense clearly makes this distinction. As we have said it intreprets all sensations, in so far as they relate to an external object, as qualities of that object and places them nearly on the same level. Moreover, the contradiction of primary and secondary qualities is no longer of any consequence whatever, since science has shown most emphatically that both were in the first place, but states of consciousness identical by nature; and in the second place, that they were only the different expressions of the action upon us of a certain cause under given conditions, and following a ratio of which one's own self is one of the terms, and the cause in question which remains to be determined, the other.

People say, "We know the primary qualities quite clearly; the secondary but confusedly." This argument in favor of a distinction had its value when science had not yet shown what could be expected of it. But now we know how to regard all those sensations formerly thought to be confused from the fact that they could not be measured, and qualities such as sound, color, odor, taste; or such others as weight, resistance, impenetrability, and in general all sensations which with the exception of extent, shape, and size were considered secondary (that is to say, confused) are at present

quite susceptible of being submitted to measurement and therefore to quantity. This progress in analysis makes sufficiently clear how slight is the foundation for a distinction uncritically established, and also how necessary it is to continually verify the content of certain notions, the frequent use of which blinds us to their significance.

III.

But how to escape from sensation? How can we pass from quality to quantity? Here indeed arises the psychological problem which interests us.

At this moment I experience a sensation of heat. What is the meaning of this fact? It means that just now my consciousness subsumes its present state under a class of past conditions called heat. Strictly speaking, I have not a sensation of heat in me. My being is simply in a certain state different from the preceding and it has the consciousness of this state and of this difference. I try to find the reason of this sensation. Now as I am near the fire I conclude that the fire is the cause of my actual condition; and then passing from my self to what is not myself, I affirm that the fire has a property, a quality, that of causing warmth.

It is important to establish the terms of the ratio: on one side, my own self; on the other, that which is certainly not my self. Two terms being present I have the right to establish the relation, and it is expressed here for me by the sensation of warmth. Have I perceived by this sensation a quality of the fire? I cannot state positively that I have; for a relation is never an expression of the existence of one of the two terms on which it rests. It is only the new result derived from a comparison of the two terms, but is neither the one, nor the other. In the above instance my sensation of warmth is but the *tertium quid,* resulting from the fact that my body is near another body which I call "fire." That is all I can say about it.

Let us continue the analysis. According to the judgment of common sense, actual sensation is what puts me into direct and immediate communication with the external world. Practically this

belief in the comprehension of external reality is sufficient. I say that I am actually warm and that this sensation is not the result of my own will. Now by repeated experiments I have come to know that fire has the property of causing the feeling which I now experience. Hence I conclude that fire has one quality (among others, doubtless) that experience has already revealed to me or will in the future, namely, that of warming; and that is enough for every-day use.

But the reasoning of the scholar is quite different. He will not draw his conclusion simply from self and non-self. He will not say: "I am warm now because the fire is warm" or "because there is heat in the fire." How could he really know it since he perceives but the actual sensation which can only give him what it implies, namely a state of consciousness for himself who experiences it, and not a certain quality of the fire, the existence of which he could only affirm positively by transporting himself momentarily into the consciousness of the fire, supposing it to have one. So the philosopher instead of concluding from a psychological state a reality outside of himself—an entity which would explain nothing—would try instead to lift the veil from the illusion of common sense. Applying the process of analysis to the actually experienced sensation, he will there establish a very nice distinction. He will first of all try to find out what this sensation within himself may be, and will say, "It is only a state of my own self; it is still my own self, but different from what it was an instant before." But here he is only at the philosophical explanation of the popular point of view; and if he did not go further with his reasoning, this consciousness of a certain psychological state which has been preceded by another, and will be followed by an infinite number of others, in other words, the consciousness of a possible series of psychical states, would be practically sufficient for him.

But this knowledge pertains only to his own being, and does not inform him of what is commonly called the external world. He is led to ask himself what it is he really knows of this state of consciousness. I say that it is a state of my being; but if I cannot reduce it to anything simpler, in what way will this knowledge help me

on with the idea I have of it? And I shall never be able to escape from the narrow circle in which the actuality of this condition encloses me, unless we suppose that spontaneously and without our being able to explain this psychological mystery, thought lays down the postulate of otherness, that is to say, something different from my present state.

Note that there is nothing excessive in the claim of this postulate. In fact we treat here the question of the psychological origin of belief in an external reality, which is found closely combined with our affirmation of the existence of qualities outside of ourselves. Now to make this task easier, we have the right to take into account all that philosophical reflection has created in the external world whether under the name of illusion or of reality. In short, we cannot neglect all the facts which actually constitute for us that kind of moral certainty which Descartes mentions. We have recourse to that well-known process of demonstration which consists in taking for granted what we are about to demonstrate. In a word, to give a solid foundation to our thesis on quality and quantity we take our footing on the notions which we consider provisionally in the light of purely moral or practical certainties and of which we shall bring out the rational character afterwards.

So, admitting the postulate of an existence foreign to mine, and which I suppose to be the cause of certain sensations for me, am I entitled to say that these sensations are qualities of an object? No, I simply affirm a relation and still have to deal only with the question of states of consciousness. In order to have a clear idea about quality and quantity, our thought must have become conscious of the objective laws which govern it. "Mere consciousness of my own existence, if empirically determined," says Kant, "proves the existence of external objects." It is necessary to understand well the sense of this theorem. I do not conceive external objects independently from my thought, but only inasmuch as they are matter necessary to its activity. My thought cannot become isolated from its external condition. In order to regard itself as a series of states of consciousness, it must take hold of something upon which to record these states. Thus we manage to affirm the existence of

an external object which up to the present time was entirely unknown, and to which we attribute as its own by a sort of illusion, qualities that are in reality nothing but our own sensations. Herewith ends the first stage of psychical processes that lead the philosopher to establish the terms of the relation between quality and quantity.

IV.

But reflection is not contented with this still confused point of view. In ancient times science found these explanations sufficient; in all things it searched only for the essence, which it identified with a certain quality. Aristotle, for instance, isolated qualities, classified them and thus settled the whole of creation in functions of one qualitative hierarchy. In the Middle Ages too, men were contented with such considerations; scholars spoke only of substantial forms, natural finalities and hæcceity.[2]

Even the Renaissance, which with its double current of scholars and mathematicians prepared the way for modern philosophy, clung to the determination of qualities belonging to things, of the forms in which they express themselves, and of the forces which they symbolize. There is nothing in that to surprise us. It is difficult for us, even now, not to interpret our internal states as real qualities of things. Why then should we wonder at the grosser illusions on which was wrecked a science which had never engaged in the study of psychical existence. Descartes's *cogito* was the signal of a reaction. While to the popular mind the growing number of experimental discoveries seemed to render more and more legitimate its belief in an external reality, to the philosopher they proved that this reality depends on ourselves since it is really known to us only when we have reduced it to a simple idea, especially that of magnitude or quantity.

The human mind knows only number adequately. By reducing perceptible things to the quantitative relations pursued by science, the horizon of thought has widened. Until then, an abyss divided self from non-self. The mind, having a conception of itself only as

[2] From the Latin term, *haecceitas*, meaning "thisness."

a mirror on which external objects are reflected, and accepting sensations as real qualities apart from itself, established on a classification of these qualities the dualism of reality, self and non-self. Since Descartes the external world is closely bound to ourselves. The illusion about quality is denounced; it is no longer of importance, but quantity (that is, numerical relation) takes its place.

With the author of *Principles of Philosophy,* mathematics is declared to be the symbol of all intelligibility and to reduce the knowledge of bodies to the laws of quantity is to bring them back to reason. The mind only grasps fully and clearly that which is reducible to the terms of ratio. Such is the nature of our thought that it can see distinctly at a glance only what it creates, so that the external world is known to it only in so much as it is the reflection of its own laws, and above all the work of the initial category of quantity from which the principles of mathematics are derived. And if since the mind has not become conscious of its conditions of existence, or in other words, since it has not understood that it can only think through analytic ideas, quality in itself is not susceptible to perfect apprehension, it is therefore reducible to something simpler than itself, and is a confused notion requiring analysis.

V.

We have seen how it is possible to understand the distinction between quality and quantity, and how the mind can pass from one to the other. It remains to show how the total reduction from the first to the second is brought about. It is the last stage in the ascent of the mind towards purely rational knowledge. This last investigation will throw a new light upon the two preceding stages of the processes we are trying to follow.

We shall see that just as in mathematics the progress of reflections consists in exhausting as far as possible the objective, concrete matter of ideas, "unceasingly diminishing the part of intuition, and seeing as little as possible in order to demonstrate better,"[3] so

[3] Milhaud, *Essai sur les conditions et les limites de la certitude logique,* p. 47.

here the progress of knowledge will be proportional to the abandoning of the perceptible element, to the predominance of representation over affection, in short, to the substitution of mathematical formulas for the purely intensive fact.

Herein lies precisely the object of physics taken in its broadest sense: to reduce the most diverse phenomena to motion, all science to mechanics. It is only a question of establishing a unit of measure for the elements of which the latter disposes, space, time, speed, form and mass of bodies in order that mathematics may adapt itself to it and furnish us with an explanation of facts. "I find," says Descartes, "that in the ideas about corporeal things one meets with very little that he can conceive clearly and distinctly, and that is to know size, or rather extension in length, breadth, and thickness, the figure that results from limiting that extension, the position that bodies of different forms hold in relation to each other, and motion or the change of that relation....As to other things such as light, color, odor, taste, heat and cold, and other qualities which come under the sense of touch, they meet in my thought with so much obscurity and confusion that I do not know whether they are real or false."[4]

The ideal of physics has not changed since that time. The question for it is still only to submit to analysis our ideas about qualities and to show that if in ourselves they are sensations, outside of ourselves they are nothing but different modifications of motion that may take place within particles of matter. Thus we must conceive the material universe as a "machine in which there is nothing at all to consider except the figures and movements of its parts,"[5] and physics in its highest abstraction is only a web of algebraic formulas, expressing forms and motions.

Then if, like M. Evellin,[6] we consider the world as a system of forces (according to the views of L'Abbé Moigno) or if like so many scholars, we study it only from a mechanical point of view, gravity, weight, mass, density, resistance, impenetrability, and many

[4] 3rd *Med.* I, 277; *Principes* IV, 203; III, 518.

[5] *Principes,* IV, 188.

[6] *Infini et quantité,* pp. 60-65.

other material properties which we transform into entities will be brought back either to the mutual attraction of dynamic monads, or to a collection of partial attractions, or to combinations of movements; but whatever the explanation may be, in the last analysis it will resolve itself into a system of equations. The mechanical nature of physical phenomena, that is the reduction of quality to quantity; this is indeed the last word of science.

It would be useless to insist any further on this point; let us rather investigate the meaning of our knowledge of things, after we have submitted them to the category of quantity.

VI.

What is a numerical relation if not a product of our thought? and why is it a product of our thought if not because laws of our thoughts are such as they are? Affirming the agreement of an external reality with a certain equation, is establishing a postulate of which nothing can warrant the foundation unless we connect it with a metaphysical idea, that of the perfect God admitted by Descartes, for instance. There may be a connection between our thought and reality; but probable as it may seem to us it is still but uncertain since in fact we only perceive through our senses, such as they are, although they might be different and since we reason by the means of what we call laws of thought such as they are but they also might be different. This starting-point admitted, the relativity of all knowledge will follow. We reduce our impressions to particular mathematical formulas, and simply by means of this reduction of a confused state to a clear idea, let us say, we manage adequately to grasp the real.

Adequately indeed, but in relation to our way of knowing. So that when we think that we are reducing external reality to a mathematical formula, it is nothing but ourselves that we are so reducing —ourselves rendering objective the product of our thought; ourselves inasmuch as the organic laws of our thought symbolize themselves, express themselves in a form we call equation for instance, and which is only a result of the laws of these thoughts beginning

to act on the occasion of any kind of a sensation the cause of which is outside of ourselves.

And what may that other thing be? Subjectively (and we are coming back to our first point of view) a sensation; objectively, the same sensation entirely transformed through thought, and this is due to the fact that the category of quantity comes into play. Objectively and scientifically, the external object is only our sensation measured in what we suppose to be its cause; it is the intensive expressed by the extensive. This expression is found moreover in the question of time and space. Space is the extended, time the unextended. We can grasp fully only the former and that helps us to know the latter; and still we get to the knowledge of space only through time, which to be measured needs a conception of space.

One of these two terms is certainly primary and doubtless it is time, since it is in time that we live, while undergoing a state of consciousness of any kind; but we are forced to admit that we get an exact conception of time only when the notion of space is given us, and also the result of the application of the category of quantity to particular sensations.

The outcome of all this is that quality is scientifically the expression of a relation between two terms of which our own self is one; objectively, an illusion derived from the habits of our mind but founded on reason, inasmuch as it is an expression of the relation established between objects and ourselves; and subjectively, a sensation whose certainty lies in our own psychical existence.

VII.

We see therefore how unreasonable it is to turn quality into a category of thought, since it is reducible to that of quantity which is in reality founded on a necessity of the human mind. Now herein consists the principal characteristic of the theory of categories.[1] They are not things-in-themselves distinct in some way from thought, after the manner of the faculties admitted by the Eclectic school. To admit them as real qualities, would be going back to the system

[1] Couturat, *L'Infini mathématique*, p. 208.

of explanation by occult powers. They are laws and still this term seems only accurate in part, for we know to how many interpretations it is liable. Whichever term we use to designate the fact that thought has states of existence, it will be impossible to determine what after all cannot be determined. Indeed we do not seize categories in the act. We infer them from all this activity of which we have only been quite conscious from the time when the results of science began to reveal themselves so considerably. Is not that the salient point of Kant's philosophy, to place science on a solid foundation? Having an immediate knowledge only of the succession of our states of consciousness, we cannot imagine in what way that wonderful structure science and society to which we all agree to grant, at least, moral certainty, has been able to build itself on the fragile basis of our ephemeral sensations. Kant attributes this to the categories of thought. This world is only my own representation, but a well-founded representation in the sense that the reality of the harmony of the external world, with the knowledge that we have of it, is all in the cognizant subject, that is in the forms of thought. Hence we see that only through induction we can become conscious of the categories; that is why we must be careful not to interpret them as realities, distinct from the sensations to which they apply, and to limit them to the simple affirmation that our thought (as well as every other thing) has its conditions of activity, which we designate by the word category.

To explain mathematically, that is, to reduce to extensive magnitude what we call the qualities of bodies, remains therefore the essential condition of sure knowledge. But if we may indeed pretend that we comprehend objects only in so far as we reduce them to the clear and distinct notion of quantity, it is not less exact to maintain that something will always remain out of the reach of exact science, we mean quality, considered as one of our inner states. Those qualities which we think belong to objects, are in ourselves; they are relations. Now being such, it is difficult for us to pretend that we can reduce them to a formula absolutely exact, for of the two terms of each relation the most important, the psychological inner self, escapes any fixed determination. The proof of it is that

those qualities which we know only from the intensity of one of our passing states experience variations according to the temperament of each individual. Quality subjectively taken always oscillates in the obscure vagueness of a more or less vivid state of consciousness.

The difficulty presented by the absorption of the intensive by number entitles us to wonder if there is no danger in wishing to submit by force the originality of our psychical life to the rigor of an equation. The same process of reduction of the external world to numerical relations raises the same objection. Do we not mutilate things by setting them in algebraic formulas?

We shall be content with saying that science does not in any way pretend to reduce the reality of things to a formula; but it simply affirms that the mathematical expressions in which it sets reality are to us the only condition of any perfect intelligibility. Thought has no knowledge besides the working of its laws. Now these laws are the principles of mathematics. When the mind subsumes its internal conditions to them it grasps in the operation all that is thinkable in the external object. That is what M. Couturat brought out clearly when he wrote:[8]

"While all perceptible qualities that we draw through abstraction from the objects of experience lose all distinctness when generalized....and give place only to vague and confused concepts, the dimensions of concrete objects thus brought out through abstraction of their perceptible qualities, keep their primitive determination. That is why natural science becomes exact only when it treats of physical dimensions and when it turns perceptible qualities into measurable and calculable qualities. Scientific knowledge could not be founded on concepts because qualities are essentially heterogeneous and hence impossible to compare with one another. To take away their original intensity and their own fine distinctions to bring them under one and the same concept, heaviness or lightness, heat or cold, is indeed destroying them, and substituting for the concrete object an undecided and colorless image, without objective or scientific worth. On the other hand science took possession of nature when it resolved to determine dimensions and their relation towards each other, because dimensions being essentially homogeneous may be compared, combined, and measured, and without losing their individual precision they may be classified under a single and universal type of magnitude. After

[8] Couturat, *L'Infini mathématique*, p. 558.

all, the concept is fatally and irremediably inadequate to the real object. Magnitude, on the contrary, is certainly not the whole object, but it is all that may be scientifically known about an object."

VIII.

Thus the problem of quality and quantity concerns at the same time: psychology, by the fact that it is in ourselves with reference to our states of consciousness that the question about the value of the two concepts arises; the criticism of the mind, inasmuch as it is important to establish the rational foundation of the transition from the intensive to the extensive by applying the notions acquired by experience to the initial category of thought; the logic of sciences, since we are compelled to found on reason the exact meaning of a term which plays so important a part in general mathematics; metaphysics, for it is impossible to do without investigating whether the two concepts have any relation whatever with fundamental existence; finally the history of philosophy, which traces for us the different points of view from which the problem of quality and magnitude has been successively faced.

To sum it up, quality and quantity are not radically opposed, except in the dominion of science; for common sense the contradiction does not exist at all. It appears only when the mind, trying to unravel the general conditions of every existence, makes the distinction between what is in itself only a *tertium quid,* resulting from the relation of things to our own selves, and what we justly call existence, which means subject to measurement or in other words that which can be subsumed under the necessary forms of thought.

Quality, scientifically speaking, is a point of view relating to the exigencies of our body in its relation to external objects; it is in us the more or less confused feeling of something good or evil for our organism. Quantity is the irreducible point of view of thought in its effort to reflect and classify the world.

JOHANNES GROS.

THE SIGNIFICANCE OF QUALITY.

THE question as to the nature of quality is one of the most important problems of philosophy. It is, first of all, of practical importance, for the difference of quality is the most significant feature of experience and in practical life it is quality not quantity we want.

The existence of quality is a fact. The question is not whether or not quality exists, but what it is and how it has to be explained. Quantity may be very important, but quality is more important, and all of us have to learn how we have to deal with it in actual life.

Quality may be (and we believe it actually is) different from quantity by being unstable and contingent upon conditions, while quantity (in so far as it is equivalent for mass) is constant and indestructible, but for that reason no one can deny either its actuality or its paramount significance. Every trader knows that good quality commands a higher price than bad quality, and quantity is only appreciated if the commodities are of equal quality.

Qualities withdraw themselves from our direct observation. They seem to be occult entities that reside within things. Moreover, qualities change. Some goods deteriorate in course of time, and on the other hand better qualities can be produced from poorer materials by human labor, be it through chemical combinations, mechanical mixture, or other manufacturing processes.

There was a time in the development of mankind in which different qualities were attributed to different agencies that had mysteriously taken possession of things; spirits were supposed to enter or to leave bodies; diseases were explained by obsession and so exorcism was the natural method of the medical art. With the

progress of science and the deeper insight into the mechanical nature of things the animistic conception changed into the metaphysical and the metaphysical into that of positivism. Scientists learned to appreciate the methods of weighing and measuring and they noticed that many differences in qualities could be explained by a difference of proportion. Thus they fondly imagined that they could discard quality altogether and have it subsumed under the category of quantity. No doubt this was possible (at least in a certain sense) in some cases where differences of mixture produce different effects, and, since quality is discredited as mystical, the assertion is made that quality is an illusion and that the old philosophers, among them Aristotle and Kant, had simply blundered by admitting the idea of quality among the categories.

In this sense M. Gros has written his essay on quality and quantity, and he proposes to explain the transition of our philosophical conceptions from quality to quantity. Modern science, he thinks, has disposed of the idea of quality and supplanted it, at least in theory, by methods of counting; but such is not the case, for in spite of the proclamation of the demise of Quality, quality will forever remain the most significant fact of experience, which, if it were merely for practical purposes, can not be dismissed and will have to be retained. A closer inspection will show that the existence of quality is not limited to practical life; it is not a mere illusion of merchants and grocers which disappears in the light of science. For theoretical purposes also the categories of quality will have to be retained as being essentially different from quantity, and this distinction will remain forever indispensable not only to the men of practical life but also to the scientist and to the logician.

Incidentally the statement should be made here that there are some scientists and philosophers who characterize modern science as mechanical and purely materialistic. It is frequently assumed that we should dispose of all former philosophies as antiquated and belonging to the animistic era of savage life. This kind of modern science has proved to be a fond illusion of the rationalist movement which was started in the eighteenth century, for it had to be abandoned on a closer acquaintance with the most important problems

of life. The rationalists of the eighteenth century overlooked entirely the existence of soul. They thought they could explain all processes by mechanical laws, but they forgot that feeling is not a motion and so could not be subsumed under the category of mechanical action. As a result Weber and Fechner introduced the theory of parallelism which showed that feeling is one side of a process which accompanies the physical actions of the nervous system. Thus they fell back on Spinoza who had been the first to teach the duality of existence which in a monistic sense he conceived as one whole, the two sides being sentiency and extension, or idea and matter, or spirit and body. In this contrast quality had been subsumed under the category of the subjective element, sentiency, and quantity under its contrast, the objectivity of extended bodies. Quality began to be suspected on account of its mysterious character. It was denounced as an illusion together with the notions of idea, soul, spirit, etc., and its non-existence was maintained. The absurdity of these theories was obvious, for the theory amounts to a negation of our own spiritual existence, of our thinking, which constitutes the very nature of our most essential being. Thus a reaction has set in and we may say that the modern tendency is no longer to discard the notions of soul, spirit, quality, etc., but to comprehend their nature and significance; to understand what they mean and how they can be explained without resorting to mysticism or a belief in occult phenomena, and this we shall attempt to do now.

Before we give our brief explanation as to the nature of quality we must remind our readers of the monistic principles of science and the philosophy of science. We must be clear as to the nature of the scientific ideas which we use as instruments of thought and we must bear in mind that all general notions are abstractions. The real world is one great totality and the observing thinker describes certain features of it which he symbolizes by a name that applies generally. Abstractions denote real and actual qualities of things but not things-in-themselves. Gravity is not an essence, not an object, but a certain feature which is observed in all things heavy. There is no whiteness in itself, but whiteness is a quality which is

observable in all things which we call "white," which possess such chemical qualities that they reflect the light that shines upon them so as to produce in our eye a chemical reaction which causes the sensation called "white." Sentiency or feeling is a quality which appears in all things sentient. All these general notions and other abstractions are methodically arranged so as to produce different hierarchies in which specific ideas are subsumed under general ideas so as to represent all things, according to rules of logic as genera and species. It is this method of arrangement which makes it possible for us to pigeonhole as it were our knowledge of things systematically and make each notion easily and quickly accessible. It enables us to know the nature of things and to do the right thing at the proper time. In fact, comprehension is nothing but an adequate description of things and a pigeonholing of each correctly under its proper category. Knowledge is a sufficient stock of such descriptions and their orderly arrangement in our minds.

As to the categories quality and quantity, we must observe that there have been things which could be counted and others the nature of which could not be established by counting, but was possessed of features that were describable only in terms referring to special experiences. For instance, we can see how many head of cattle there are in an enclosure, we can tell how many pounds a bag of salt weighs, we can measure how high a tree or house may be, but we cannot by measuring or weighing set forth the taste of salt; we cannot by measuring or weighing explain the nature of a circle. Of course we can measure the size of it, but in order to describe the figure of it we must draw a circle and show it. No amount of counting or weighing will explain its curvature or the relation of the circumference to the center. Thus quality is a thing that stands in contrast to quantity. It cannot be determined by measuring or weighing, but is in need of special experiences, and those who use the same language know exactly what is meant when we speak of the taste of salt, or the taste of sugar, or the color sensations of red, green, blue, etc.

Some sensations have been reduced to a difference in quantity, for instance: rough or smooth may under circumstances be due to

more or less high elevations on the surface. The smoothness of velvet is due to a fine fibre of its fabric which may be of a definitely measurable thickness and height. The rougher plush may be similar in its manufacture, only showing the thread thicker and longer. Thus we may in certain cases reduce quality to quantitative measurements and say that the feeling of roughness or smoothness depends upon certain arrangements that are quantitatively determinable; but this is not true of quality itself, only of some features of quality, and assuredly it is not true of all qualities. Some qualities are due to a difference of configuration. Opaqueness of a piece of coal and the transparency of a diamond are due to an arrangement of their atoms, and we have here a difference of quality which cannot be reduced to a quantitative analysis. It is a difference of quality. The material of which both bodies consist is the same, but the arrangement is different and is due to form.

Difference of form is describable but it cannot be determined by the use of either the yard-stick or the scales. Its nature cannot be defined by either measuring or weighing.

What is true of the qualities of the diamond and the coal is true of all other qualities, spiritual as well as corporeal. Nor is it true that quality is restricted merely to the subjective sphere of existence. Quality plays also a most significant part in the objective world of bodily extension, and indeed the difference of psychological qualities will find its explanation according to the theory of parallelism in the physiological formation of its bodily counterpart. The difference of red sensation and white sensation can be explained by a difference of the nervous state of the retina, and thus an insight into the physical laws of our nervous structure will throw light upon the psychical process of our soul-life.

Intimately connected with the several problems of quality is the idea of unity. In fact the two notions, "quality" and "unity," appear to be inseparable. Every unity is possessed of a quality of its own, and whenever by combination a new unity is created, we are confronted with a quality which originates not as the product of a quantitative summation of its elements, but through a characteristic interaction of parts.

A clock, a steam-engine, a dynamo, etc. are not quantitative amounts of metal, but definite configurations of wheels and other contrivances of machinery which perform a certain kind of work. In addition to the sum total of the weight of all particles a new value is established which is of a qualitative nature; so that in case the machinery breaks down, although there is no loss in quantity, that imponderable something which constitutes the nature and usefulness of the machine, its quality, is gone.

Every chemical combination is a new thing that acts otherwise than did its several constituents.

The qualities of salt (NaCl) cannot be explained as the sum of the qualities of Natrium (Na) and Chlorin (Cl). The taste of salt is contained in neither of its constituents; and so all other qualities of chemicals originate through combination according to the way in which they are grouped.

When we draw three different straight lines in three divergent directions and prolong them until they meet, we produce a triangle; and a triangle is a new geometrical figure with definite angles, possessed of a constitution of its own. The nature of a triangle cannot be deduced from the nature of the several lines; the triangle is a configuration representing a new unity, possessing qualities not contained in its constituent parts.

The combination of parts into a higher unity produces effects which are not a sum of equivalent elements, but introduces a new factor which is of a qualitative nature.

Quantitatively, the universe remains the same, and in every process of nature we can trace the same amount of matter before and after each change. Qualitatively, the universe changes. New qualities originate and old qualities disappear. There is no increase in either matter or energy, but there may very well be an increase of value in quality, or, *vice versa*, a loss. The effort of life is everywhere directed toward a favorable change of quality so as to transform the materials on hand into goods that will give more sustenance of life, greater comfort and better facilities.

And what is the part played by unity in this transformation?

If a number of grains of sand are heaped up in a pile, we have

a mere summation of their several qualities; nothing but an addition of their weight, mass, etc. But if a number of constituent parts enter into a compound which forms a higher unit we produce something new that did not previously exist. A unitary complex is not merely a summation of its constituent parts; it contains a new factor which is not of a quantitative but a qualitative nature, originating through the co-operation of its parts; and this new factor would not have been produced by any of its parts alone, but is the result of their mutual interaction.

The characteristic features thus originated are sometimes most marvelous in their effects and thoughtful observers of natural phenomena, accordingly, have been overwhelmed with awe and wonder. The origin of higher unities with their production of new qualities of most astounding and unheard-of effects, has become the main source of all mysticism; and, assuredly, the inexhaustible possibilities of new formations are the most fascinating events of life, sometimes as surprising as the tricks of sleight of hand.

Supposing the world to exist of homogeneous material of some kind whirling about in cosmic space, we find it consolidated in the alembic of nebulæ into hydrogen, oxygen, nitrogen and other chemical elements, which condense into planetary systems. Every chemical element is not merely the sum total of a certain amount of world-stuff, but a new combination in which, through peculiar groupings of its particles, new units of interaction are created that possess an individuality of their own with qualities that did not exist in the homogeneous ether. Higher complexes are formed in more complicated chemical combinations, and inventors are busy to increase their number by artificial formations. Some chemicals crystallize. According to their angle of juncture they combine with substances of the same kind in different geometrical forms of crystallization. Every crystal forms a new unit, and though its elements are homogeneous in their chemical makeup, yet the crystal as a whole is something new which as such did not exist before.

The most important new departure is attained by organization which produces the phenomena of life. Some of the most unstable chemicals enter into such a co-operation that they form a constant

circuit of materials which by constant renewal of its waste products preserves the original form of the same round of function, and this process is called metabolism. It is a combination of adaptibility and permanence; the old form is preserved but constantly modified by new experiences. We call the new product of this wonderful interaction of parts vitality, and there are two forms of it; one, endowed with sentiency, and the other void of it; the former develops into animals, the latter into plants. There is a qualitative difference between the two, and each is differentiated into higher and more complicated forms in the course of evolution.

We see that we can not appreciatively study nature or understand the evolution of life without comprehending the significance of quality. In quality the creativeness of the universe reveals itself, and in contemplating the increasing efficiency of quality, we are confronted with the divinity of nature.

Every unitary complex is an individual, and the tendency is that unitary complexes in their turn will combine into higher unities. The higher unitary complexes in the domain of life are called organisms; their parts which in themselves are smaller units depending on the co-operation of the whole, are called organs. Organs are such intermediary units as could no longer lead a separate individual existence. Thus they are, in a certain sense, true unitary complexes, but are never found separate or isolated.

The highest kind of a unitary complex is man, who as such is called a person. Several persons enter into higher combinations such as states, churches, societies, corporations, etc., and even their interrelations coalesce into organized institutions such as language, habits, laws, forms of government, etc.

Properly speaking, all persons are organs of the social whole; for it will be seen that the combinations of personal interrelations are the most important features of human life. If we could cut out from a person all he owes to society and his social intercourse with his fellow-men, nothing would be left of his personality but the mere physiological organism. Our exchange of ideas, our school institutions, trade, commerce, and even our charities, also the struggle in our social life, competition, trust formations, labor unions, the

antagonism between capital and labor, etc., are due to new combinations in the field of human hopes and ideals. The interrelations of human society are being constantly readjusted, and the result is a difference which is not purely quantitative but qualitative.

The problem of unity found a peculiar construction in India where the unity of the soul was hypostatized into a thing-in-itself called *atman* (or "self") that was assumed to be independent of its parts. The Vedanta affirms, and Buddhism denies, the existence of an atman. Buddha proclaimed as an essential truth of his doctrine that (1) all compounds are transient, (2) that all compounds are subject to suffering and (3) all compounds are lacking an atman, i. e., a self that is independent of its parts. The Vedantic view leads to mysticism while the Buddhist doctrine takes a bold stand upon a sober and purely scientific conception.*

There is no need of entering into further details or pointing out all the applications of the non-existence of the atman, but we must insist on the paramount importance of the part which unity plays. Though a unitary compound, be it in the shape of chemical molecules, or organisms, or inventions of machinery, or mathematical concepts, or ideals, is not a thing-in-itself, though it is not a metaphysical entity which owns its parts as if they were its properties, though there is no atman; yet the effects of a unification are real, and so we say in a certain sense, that things act as if they were ensouled with atmans.

Atmans are conceived in the Vedanta as eternal and immutable, not subject to time and space, and the same can be predicated of the unity of compounds without ascribing to them any mysterious selfhood of atman-existence. Though unifications must be accomplished in time and space, they are possessed of a pre-existence as potentialities in the womb of eternity. They are the Platonic ideas which constitute the formative factor of existence. They are not things-in-themselves but forms-of-themselves. Like pure mathe-

* How difficult the question of unity proves to be can be seen in Plato's attempt at solving the contradictions of "the one and the many." Concerning Kant's theory of things-in-themselves see the author's book *The Surd of Metaphysics*, pp. 6 ff., and 29 ff.

matics, they exist in the domain of potentiality, and their realization, unlike the invention of a fairy tale, has not been woven out of the pure fancy of the inventor. Their realization is, properly speaking, an incarnation or actualization of eternal possibilities. In the same way the invention of machinery, of the wheel, the sewing-machine, the steam-engine, the dynamo, the motor, electric light, etc., are (as indicated by the word) in-ventions, i. e., findings; they are discoveries; in fact they are revelations of hidden truths; they existed as much before their invention as did the continent of America before its discovery either by the Norsemen or by Columbus. They lay concealed in the unfathomable abyss of the laws of being and no one knows what wondrous surprises the future still has in store for us.

Thus we find that the old contrast of quality and quantity is justified. The old philosophers who distinguished these two categories cannot be branded as dupes of an illusion, and we would make a great mistake if we discarded these notions or tried to obliterate the idea of quality. In fact far from rendering our insight into nature clearer it would obscure matters and would involve us in untold contradictions. On the other hand the suspicion which we rightly entertain against notions that lead to mysticism has been disposed of. Quality is by no means an idea which necessarily leads to occultism. A right conception of the nature of quality, which is always due to a definite configuration or structure, constituting a higher unity endowed with new and characteristic features of its own, not contained in any one of its several parts, shows us that there is as little mystery in the differences of qualities as there is in counting, in weighing, and in measuring.

The significance of our solution can hardly be underrated. It throws light upon all problems of philosophy, including the domains of psychology and ethics. It is a solution which commends itself by soberness and justice,—soberness because it disposes of the mysterious aspect of quality without denying the remarkable facts that naturally give rise to mysticism, and justice because we recognize the truth in the statements of both opponents; on the one hand those who insist on the significance of quality as a fact of experience

which is true though it might smack of occultism, and on the other hand those who endeavor to reduce all phenomena of experience to a quantitative analysis in the hope of rendering the universe as intelligible as any system of mechanics.

There is no use denying the wondrousness of thte facts of experience after the fashion of the old rationalists, nor is there any need of seeking refuge in agnosticism. The appearance of new qualities necessarily seems a miracle to the uninitiate, and even when we explain these occurrences as the inevitable results of definite combinations of parts into new unities, the fact remains as strange as before. On account of it the universe we live in is replete with illimitable potentialities, a condition which constitutes the main charm of life.

The nature of our own being—the human soul—stands out foremost among all the qualities that challenge the curiosity of the investigator; it is the noblest phenomenon of the universe, and the deepest problem of science. The qualities that appear in the domain of psychology and ethics, constituting the background of religious life and the history of mankind are the most wonderful facts —more wonderful than could be invented by any poet or romancer. We have good reason to assert that all of them are explicable by science, and yet they remain what they are—wonderful, curiously wonderful; apparently miraculous, and certainly divinely grand.

Clearness of comprehension does not destroy the worth of things, and an explanation does not dispose of the facts explained.

EDITOR.

THE SEARCH FOR THE PROPHETS.

THERE is no part of the Old Testament that has for us greater interest than the prophetic literature. In its impassioned poetic passages, in its lofty ethical thought and in its profound interest in life, it is far superior to any other type of literature which we find in the Hebrew canon. This is but saying that the greatest minds in Israel expressed themselves as spokesmen or interpreters of Yahveh in oracular literature, the highest thought was molded into prophetic forms of expression. For this reason the question, who wrote this literature? who were the men who cast their thought into oracular molds? is an important critical question to the student of Hebrew life. The question has the greater significance, it is worthy of note, because the prophetic literature is so extensive. One fourth of the Old Testament canon consists of prophetic books and pieces. Three of these books are individually equivalent to a duodecimo of one hundred and fifty pages. This prophetic literature is indeed, for an ancient literature, of vast dimensions.

It has commonly been supposed that the writing prophets are known to us through the titles which the various books and prophetic pieces bear and through occasional personal allusions in these writings. There has been little disposition to question these titles and allusions; especially as they have been thought to be supported, measurably at least, by references to these prophets in the historical books; but such references are so few and so suspicious as to their character that the search for the men themselves becomes to him who prosecutes it very perplexing. Apparently too much has been taken for granted. It has been assumed that just as we

know certain Greek tragedies as the work of an Æschylus, and certain as the work of a Sophocles, and certain others as the work of an Euripides, all prominent in Greek life; so we know the various prophetic books and pieces of the Old Testament as the work of the men whose names they bear, which men indubitably appear in the historical annals of Israel. Such, as a careful critical study reveals, is not the case.

The Greek tragic poets find a large place in the life and historical annals of the Greeks of the fifth century before Christ. Æschylus is known to have borne an honorable part in keeping the Persians in the Marshes of Marathon and to have aided later in the overthrow of Xerxes at Salamis. It has been said of him that he was honored by his countrymen as a patriot rather than as a poet, though as such for a generation he was awarded the prize for superiority in the drama. Sophocles is known to have led as a youth the chorus which danced and sang around the trophy in celebration of the battle of Salamis and to have served as a colleague of Pericles in the Samnian war. The statement of his biographer that he bore his full share of the civic burdens of his people and that he served frequently in foreign embassies, has been regarded an exaggeration. There can, however, be no question but that as a poet for three decades he was prominently before his people and kept in close touch with the great movements of his time. Euripides had less fondness for public life; but his work as a dramatist kept him before the people for half a century while his unhappy family life became notorious. Though he in his old age expatriated himself, it is said that when the news of his death reached Athens the whole city was thrown into mourning. The same is not true, so far as we can discover, of the writers of the Hebrew prophetic literature. They do not appear in the historical annals of their supposed time.

To him who is interested in the life and thought of Israel there is no more fascinating diversion than the search for the literary prophets. It is upon this still-hunt that I purpose to take my readers. My reasons for so doing will appear later. It is enough at this point to remark that our interest in the ethical thought of the

past—and nowhere is there to be found prior to Jesus's time loftier ethical thought—leads us to desire to place this thought where it belongs, that we may appreciate it and grasp its significance to Israel and to the world. If we date some of these prophets two or three centuries too early, we must think of them as belaboring their people with messages which they could not understand, and which, because they were untimely, accomplished nothing. If, on the other hand, we date these writings where they belong, we may be able to determine what their authors accomplished as the great ethical teachers of their time. This will remain true though we may find that these books and pieces were written by men who belonged to a literary school and did not exercise orally the functions of prophets.

In our search for prophets who are thought to have given utterance to oracles which were afterwards written out fully, or in part, we confine our attention to the books of the so-called major and minor prophets. The supposed authors of these only are known as literary prophets; and it is for traces of the literary prophets we are looking. We cannot forbear noticing at the outset that there are credible reasons for believing that there were prophets, men who professed to speak for Yahveh in ancient times among the early Hebrews, just as among other peoples there were men who stood forth as prophets and professed to speak for the gods of their people. Scholars have been wont to regard suspiciously the words put in the mouth of Nathan in 2 Samuel (see vii. 2 ff.; xii. 1 ff.); but we may take the allusions to him in 1 Kings (i. 8 ff.) as revealing the existence of an actual prophet who was a member of David's household. His functions may have been few and simple; and he may have been wholly subservient to his royal master, still it is reasonable to suppose that David had some such man who professed to be able to ascertain the will of Yahveh and to speak for him.

There are numerous allusions in the history of the kings of Judah and Israel to such a class of men. Ahijah the Shilonite who is said to have encouraged Jeroboam appears to have been such a one (1 Kings xi. 29 ff.); though the Deuteronomists spoil the

simple story told of him by their expansion (see vss. 32 ff.). It is not unreasonable to suppose that Elijah and Elisha, despite all the legends which gathered about their names, were men who figured prominently in certain directions in the life of Israel, though the latter seems to have been more inclined than the former to play the part of a courtier. We are told that Jonah, son of Amittai, served at the court of Jeroboam II, and that he encouraged him in his imperialistic policy (2 Kings xiv. 25). Such, not to mention others, were the prophets of Israel and Judah. They were men of action, rather than men of letters. There is nothing to lead us to suppose that they ever committed such petty oracles as they uttered to writing, if, indeed, they were possessed of the art. Our search is not for such men; but for traces of the men who are named as the great actors of the prophetic literature, who are said to have uttered the great oracles which were grouped about their names. Can we discover in the actual history of those times such men as Amos, Hosea, Isaiah, and Jeremiah? For these men we are to search. If they were actual personages, and, especially, were the great voices and actors of their time, then we should expect to find many traces of them in the annals of those old days.

We have to confess at the outset that we do find, as, indeed, we have seen, mention of a certain Jonah ben Amittai; but we hasten to remark that this man who figures in the book of Jonah, as scholars have long since noted, was imaginatively taken by the late writer of the book as his supposed prophetic actor. Some four or five hundred years must have separated the age of the prophet who frequented the court of Jeroboam II from the time when this delightful little evangel was written. Here, then, we have, admittedly, a late piece of literature, dated back and fictitiously assigned to a prophet of the eighth century. A study of the phenomena which this book presents raises in our minds the question whether this is the only pseudepigraphic prophetic piece in the Old Testament. It would be strange if this were the only one, so strange that we note with interest the fact that other prophetic books have been found perplexing, that scholars have been wont, for example, to question whether Malachi was an actual prophet, or whether

the name was that of the supposititious prophet of the writer who had given Israel this little book that is universally recognized as late. It is not matter for wonder that they should have done so, for the name means "my messenger." It is just such a name as we may conceive a writer who wished to put his thought in prophetic form might have taken. Of this Malachi there is no trace in the history. "Obadiah" (servant of Yah, or Yahveh), was a common name among the Hebrews, but of a prophet of this name the history gives us no hint. The oracle against Edom which forms the burden of the little prophetic piece, which is ascribed to him, might have been published near the Maccabean age; for we know it was not until the time of Judas that the Edomites, who had been crowded by the Nabateans up into southern Judea in early exilic days, were subjugated (1 Maccabees v. 3 ff., 65). The oracle is really an expansion of one which occurs in Jeremiah (xlix. 7 ff.). As for the name, here again we have just such a name as a late writer might have taken if he wished to conceive of an oracular prophet as uttering what he had written.

Nowhere in the historical books do we come upon prophets known as Nahum and Habakkuk. Nahum (consolation) is a fit title for the supposed prophet of the impassioned oracle which, Otto von Happel in his recent handbook thereon concludes, must have been late, thus agreeing substantially with my unpublished notes of two or three years ago. There can be no question but that the allusions to the Assyrians are veiled allusions to the Greeks or the Syrians, while Nineveh stands for some city of the third or second century B. C. As surely are the references to the Chaldeans in Habakkuk veiled allusions to later peoples.

Of a prophet known as "Zephaniah," who appears as the preacher in the little prophetic book which is said to be his, there is no mention in the history. The manifest attempt to give this supposed prophet a royal pedigree in the title renders the existence of an actual prophet of this name all the more doubtful. Of Hosea, Amos, and Joel there is not the shadow of a trace in the history of Israel. This is, as scholars have confessed, remarkable. Amos, as the herdsman of Tekoah, who is thought to have gone north

as a Judean prophet and roared, though ineffectually, at the old sanctuary of Bethel, has been declared to be "one of the most marvelous and incomprehensible figures in the history of the human mind." So incomprehensible has he seemed that we can understand why one scholar has been moved to remark that Amos could not have gone north to the Bethel of which we read in North Israel, but must have gone rather to some house of God or local shrine of that name in Judea. Hosea is thought, as the title of the book named as his intimates, to have labored as a prophet in the north for half a century, though no trace of him is to be found. The book of Joel comes before us as a problem quite as perplexing. It is significant that the book of Amos which is a burden, a message of doom to Israel and Judah and neighboring peoples, with but a word of bright promise and assurance at the end thereof which has been supposed to be an appendix by another hand, has for its title "Amos" (a burden), a name which occurs nowhere else in the Old Testament. While "Hosea" and "Joel" are common Israelitish names, it is easy to see why they may have been taken by late writers as names of the authors of their prophetic pieces.

More astounding still is the fact that Ezekiel, who is supposed to have been the great prophet of the early exilian days, is mentioned nowhere in the Old Testament outside the book which bears this name. Even in the book the name appears but twice. On the very reasonable supposition that the book is a late pseudepigraphic work whose author purposely dated it back several centuries, we may explain the failure to speak of such a one as Ezekiel in the historical annals, not otherwise. We are likely to find that Zunc and Seinecke, as they labored long ago in this field, were the pioneers of a more intelligent conception than were their opponents.

Micah the Moreshite is named as the prophet of one of the short prophetic books. Though unmentioned in the history, this supposed prophet is quoted by name in Jeremiah (xxvi. 18). This has little evidential value without Jeremiah as a prophet actually played an important part in the life of Judah in the days of Josiah and later.

Haggai and Zechariah who are named as the prophets who gave

utterance to the oracles of two of the minor prophetic books are mentioned in Ezra (v. 1; vi. 14); but if we are to conclude with one of our modern scholars that Ezra was, as he appears in Hebrew literature, a creation of the late priestly school and that the book which bears his name was largely, if not wholly, imaginative, we must admit that the mention of such prophets in the late literature counts for little. In some curious ways the book of Ezra and these two prophetic books are interlinked. It is possible that both Haggai and proto-Zechariah were written to bolster the fictitious priestly story of the return and restoration which we find in Ezra. We can safely assert that in the actual history of Israel no such prophet appears.

Two prophetic books remain to be considered, two of the longest and most important to the student of this part of the Old Testament literature, Jeremiah and Isaiah. There is probably no one of the literary prophets who has puzzled scholars more than has Jeremiah. According to the book which bears his name he must have been the most important personage in the last half century of Judean life prior to the exile. In the days of Josiah and for several decades thereafter he is said to have figured prominently as the spokesman of Yahveh. He is pictured as throwing himself into the civil affairs of his time with a zeal which knew no abatement and with a heroism which was unquestioned. Opposed, yet never disheartened, thwarted and made to suffer terribly, yet never crushed by adversity, he made his influence powerfully felt in a lost cause. Recognizing the inevitableness of the spread of Chaldean power he unweariedly endeavored to save from utter ruin the Judean state and to turn the thoughts of his people to the loftier ethical ideals for which they might live as dreams of material advancement and splendor faded. Such he appears to have been, according to the writings and the oracles incorporated therewith, attributed to him; yet when we come to search for him in the history of his time we discover not the slightest trace. This, as Dr. MacCurdy and others have noted, is marvelous and manifestly inexplicable.

It must of course be admitted that the references to Jeremiah in the late priestly literature (2 Chron. xxxv. 25; xxxvi. 12, 21, 22;

Ezra i. 1) are without critical value, as is the mention of the name in Daniel (ix. 2), a Maccabean book. Such allusions were made long after, though perhaps not more than a century after, the writer of Jeremiah had published his work and it had been accepted as an ancient prophetic book. When we call to mind the fact that some three or four centuries intervened between the date usually assigned this supposed prophet and the date of the first priestly mention of him we can easily see how little warrant we have for assuming that there was an actual personage of that name in Josiah's day. We have also to take into account the fact that the writings promulgated as Jeremiah's are thoroughly saturated with Deuteronomy and must in consequence have been written some considerable time subsequent thereto. If the Deuteronomists were post-exilic, as we shall have to conclude they were, then the book of Jeremiah must be placed somewhere between 400 and 250 B. C., or even later.

In turning to Isaiah, we are interested to notice that much of the book has long been thought to be the work of an unknown exilic writer who for convenience is called Deutero-Isaiah; even much of what was once thought to belong to the original, or proto-Isaiah, is now, for reasons which appear to be conclusive, assigned to the second Isaiah, or some later prophet. To some, as to Canon Cheyne (see *Isaiah, S. B. O. T.*), but little is left of the work of the first Isaiah, the equivalent of about fourteen chapters in all. The mention of Isaiah in the titles of chapters i and ii has no evidential value, for either we must consider the titles late, or must, and more reasonably, with certain advanced scholars, regard these chapters as a part of the work of Deutero-Isaiah, and therefore late. Similarly the mention of Isaiah in xiii. 1, the title of an admittedly late oracle against Babylon is equally valueless. Let no one remind us of the Assyrian chapters (xxxvi-xxxix) in which the name of this supposed prophet occurs several times, for these are undeniably late, as Dr. Cheyne has conclusively shown. To discredit these chapters as the work of an original Isaiah is to throw out 2 Kings xix, xx as evidence that there was such a person in the days of King Hezekiah. The mention of Isaiah in the Berodach-

baladan episode (2 Kings xx. 12-19; Isa. xxxix) may form a possible exception, though it is not wholly free from suspicion. It certainly is somewhat after the character of the accredited stories of the old prophets of action, unlikely as it is that such a prophet bore such a suggestive name as "Isaiah."

There remain only the references to the supposed prophet in chapters vii and xx. The passages in which these allusions to Isaiah occur are quite different from anything else which is found in this book, absurd and apparently incredible as they are. In the first passage we are told that Isaiah at the command of Yahveh went forth to meet King Ahaz with his son whom he had strangely named "a remnant shall return," and encouraged him with fitting words. We are expected to believe that Isaiah, about a century and a half before Jerusalem fell and two or three centuries before the pious Zionists began to straggle back from foreign ports, by this wondrously suggestive symbolic name thus designated his son. The story is palpably absurd: it puts too great a strain upon our credulity. And a symbolic name may have been given by a late writer to a son of his supposed prophet very naturally, for then did he and others cherish the hope of a return and of a resurrection of the old Hebrew state, then and not until then.

Quite as incredible is the allusion to Isaiah in chapter xx. According to this passage Isaiah is said to have gone naked and barefoot about Jerusalem for three years as a sign and warning against Egypt, lower and upper (Mizraim and Ethiopia), thus insinuating to his friends and neighbors who were said to be looking to Egypt for help, that the people of that land were to be led abroad as captives, and that dependence on them was therefore vain. Something akin to this, the old prophets were ever doing; but it is hardly conceivable that the writer is here describing an actual occurrence.

We are thus forced to conclude that neither in the historical books nor in this great prophetic book do we come upon convincing evidence of the existence of such a one as Isaiah. It is significant that the very name "Isaiah" is symbolic, and is strongly characteristic of the oracles throughout. It is the salvation or deliverance of Yahveh which is the theme of the book; and "Isaiah" means

"salvation or deliverance of Yah, or Yahveh." Here, moreover, as in the case of other supposed literary prophets the name of the supposed father of the prophet is of one who appears nowhere in the history. Besides "Amos" means "strength," or "strong one." Even if our search in this instance be thought to bring to light an actual prophet of the olden time we are not to think of him as having anything to do with the oracles here brought together. We may regard them as late writings thrust back imaginatively into the earlier time by their author or authors in a way thoroughly characteristic of the Hebrews.

We shall have to admit that our search for the literary prophets of Israel has not brought such men to light. Nowhere do we find traces of such men. Though they are pictured in their supposed writings as figuring prominently in the life of Israel from the close or middle of the eighth century on for five hundred years; we cannot in the chronicles of the people put our hands on them: always and everywhere they elude us. Need we wonder that scholars who have spent much of their time in this quest in their study of the prophetic literature should have drawn back amazed and perplexed?

I ask no one to accept my conclusions; but I cannot forbear saying that I see but one reasonable explanation of this most perplexing problem, it is that in common with most of the other Old Testament literature the prophetic is pseudepigraphic and as such is late. Such writings must be accounted for. There is in the Old Testament nothing finer, nothing more uplifting from both the poetic and ethical point of view than large sections of the prophets. For good reasons the great men of the post-exilic time chose to publish anonymously or pseudonymously their prophetic thought. Taking up the work and following on after the Deuteronomic school, the school which for convenience we call the prophetic, chose to work out of sight, giving an air of mystery to their pieces as they were sent forth; as the publication of Deuteronomy itself had been compassed about with mystery, as a work attributed to Moses, the reputed discovery of which in the time of Josiah was said to have occasioned a great and drastic reformation.

If my findings as to the Deuteronomists are accepted there need

be no question as to the prophetic writers that cannot be reasonably explained. If we can conceive of pious Zionists as they straggled back from foreign ports with their nobler conception of Yahveh and their purer ethics in their endeavor to build up a new Israel on the ruins of the old, labored as Deuteronomic monotheists, redacting after their peculiar manner the old chronicles of their people and promulgating Deuteronomy as a recapitulation and enlargement of the Book of the Covenant, as passing over into the prophetic school which was further reinforced by other returning Zionists, we have a conception of the prophetic literature which is most inspiring as it is withal most reasonable. Already the discovery of the fact that Deutero-Isaiah is post-exilic and late, that the burden of its matchless oracles has to do with the *golah,* the Jews who returned from Babylonia and other distant parts, has rendered it a most fascinating book. A more careful examination of the prophets reveals the fact that large portions of them are of the same general character as Deutero-Isaiah, that whenever the *golah* is mentioned in the prophets it is with radiant optimism. To think of the prophetic literature as late and to discover how the writers of this literature turned to the Zionists to whom they undoubtedly themselves belonged is to find how needless it is to mutilate the prophetic writings and assign to late writers all portions and fragments which are hopeful and optimistic.

We cannot of course be sure of the integrity of any one prophetic book; though we may seriously doubt if it be any such patchwork as some find such books as Amos and Isaiah to have been. Still there is nothing inherently incredible in the thought, if these different books are late, that they are for the most part the work of men who gave their books an individuality and unity that may be apparent to us; while certain of the books may be the work of two or more writers. In a few instances a prophetic writer may have produced more than one prophetic piece or may have had a hand in the composition of two or more of the larger books. There are chapters of Jeremiah and Ezekiel that may have come from the hand of the writers of the more optimistic portions of Isaiah. It is thus that a prophetic school sends forth its work. Once grant

that the prophetic literature of Israel is the pseudepigraphic and late work of such a school and we have to admit that the question of the authorship of any individual book or piece is of no particular moment as it certainly is as much beyond our power to determine as the authorship of a psalm. It is significant that the titles of so many of these prophetic books have been regarded by certain of our great critical scholars as spurious. Taken as they stand they are supposed to be inexplicable. By looking upon the prophetic writings, as I have here suggested we must, we may let the titles stand as the efforts of their late writers to give an imaginative setting for their work in earlier centuries.

EDWARD DAY.

NANTUCKET, MASS.

CONFLICT OF RELIGION AND SCIENCE.

FROM A JAPANESE POINT OF VIEW.

THE century which closed a few years ago must be regarded as an eminently successful period for science, whose influence is everywhere felt at this beginning of the twentieth century. Japan, too, which has been following in the wake of European scientific progress only during the last few decades, could not but feel its consequences. Men of science have shown their brilliant achievements in almost every phase of life, and naturally they stand now before us much elated over their triumphant march. Japanese education, which had hitherto placed too much importance upon ethics to the exclusion of scientific study, could no longer hold its ground against this general assault so successfully carried on, and a reaction soon set in. People ran wildly after things scientific, applauded the scientific frame of mind, and showed everywhere a disposition to disregard the old traditions. Recently, however, symptoms are becoming more and more recognizable, that point to a revival of the old spirit. Young men of Japan are beginning to feel that science does not necessarily satisfy all their moral needs, and that it is a mistake to attach too much significance to it. But I am seriously in doubt as to whether this revival really is to be welcomed at this juncture in the development of our national culture. By no means do I wish to make light of the ethical culture which was strenuously pursued by our fathers. The point I wish to make, is that this is not yet the time to revive ancient tradition at the sacrifice of the general scientific culture which has but lately

[1] Translated by T. Suzuki.

started. A universal propagation of the scientific spirit among our people is still needed, and even more urgently than it was a few decades ago.

In this connection, I have one or two questions to ask: "Do science and ethical culture antagonize each other or not? And further, not only do they perhaps not antagonize, but do they not actually coöperate with and complement each other?" Broadly speaking, science aims at discovering and elucidating laws that govern natural phenomena, while by culture we mean the improvement of our subjective attitude toward the laws thus formulated, or in short, character building.

When viewed from this standpoint, science and culture must be said to complement and not to contradict each other. Why, then, are some people inclined to think otherwise? The explanation of this lies in the fact that such thinkers are generally specialists who, in the heat of enthusiasm over successes achieved in the particular department of science to which they have devoted their lives, forget the necessary limitations of their subjects and arrogantly attack hypotheses and even ignore facts, when both facts and hypotheses are beyond the scope of their own study, and, therefore, can not be included within the bounds of their favorite theories.

The proper object of science is to investigate various laws controlling natural phenomena. For instance, it is a proper subject of scientific study to find out the laws that govern the phenomena of mutual attraction between the sun and the earth, or those which make one element combine with another in various degrees of affinity, or to discover those psychological principles which prompt men and women to unite themselves in matrimony. Sciences, to use logical terms, are made to determine some particular propositions, explaining each natural phenomenon by a certain definite law. They have no right whatever to overstep this limitation and absolutely deny or affirm things which do not properly belong to those specific spheres.

Suppose here is a man whose special study is psychology. He can investigate the relations that exist between the soul and the body, between the nervous system and consciousness, and he can

assert that particular states of consciousness are accompanied by or follow from corresponding organic or nervous conditions; or, that whenever the brain is affected in a certain way, a certain state of consciousness may be expected. But he will go beyond his sphere of study, if he tries to positively decide the problem whether our consciousness could continue to be active after a dissolution of the material body. The psychologist, who studies the relation between the mind and the body, feels naturally disposed to think that without a nervous system mentation would be impossible. But he, as a scientist, has no right to make, or rather has no positive facts to support, the unqualified denial of mentation without cerebration.

Let me make a wider application of this scientific attitude and method. Our wonderful solar system is, as is well known, mathematically governed by the laws of attraction. But we cannot say by reason of these laws that there is no God in the universe controlling its multitudinous phenomena. We are not scientifically justified in making such a universal negation. In this respect Newton was quite right. He has scientifically—mathematically—established in his *Principia* the laws of gravitation, by which all celestial bodies attract or repel one another. But the fact did not warrant him in advancing his logic further, and denying the existence of God. On the contrary, he said that there probably is a supreme being above all these natural laws.

Unfortunately, however, a number of scientists both in Europe and America, dazzled by successes brought about by some special scientific investigations in their chosen spheres, have sometimes yielded to the temptation of unscientifically extending their conclusions beyond the limits which they have set for themselves. Thus, even the principles of ethics and religion have been discussed and attacked and condemned by the scientists, whose business, strictly speaking, had nothing to do with those subjects. Rising against these encroachments, philosophers and men of letters declare that there are some facts and principles which defy treatment by regular scientific methods, so called. The sudden revival of mystic romanticism whose waves have also begun rolling over the Japanese sea

of thought, may be considered as a reactionary movement against the belief in scientific omnipotence.

Whatever its claim, mystic romanticism has at least one lesson to teach us. This is its peculiarly inspiring stimulus to our inner life. According to its doctrine, the human soul is considered to perform three distinct functions, thinking, feeling, and willing. Each of them has peculiar needs of its own. Science and philosophy may satisfy our intellectual demands, but utterly fail to administer to our emotional needs; and what mystic romanticism strives after is to fill this deficiency. As far as it stands above simple mechanical intellectualism it has a unique fascination, which strangely appeals to man's innate spiritual aspirations. It cannot be regarded as an abnormal pathological phenomenon of the soul, for a mystic element is surely to be found in our normal mental activities. Its only danger is that some weak-minded people are apt to take refuge in mysticism, thereby excusing themselves for want of thought and giving a free rein to their unjustifiable subjectivism.

I cannot here enter into any detailed discussion concerning the recent revival of mysticism in Europe. But we can remark that the further the progress of various sciences, the more complicated, detailed and specialized they become, and this fact finally tends to isolate the scientists of one branch from those of every other, confining each within his own narrow department. Therefore, the social soul, if there really be such a thing, at this beginning of the twentieth century, must be said to be losing unification and gradually disintegrating. In this sense we can say that the present social soul is pathological. Such men as Helmholtz, Virchow, Huxley, and Tyndall, are getting rarer every day. They were scientists of a high order, and at the same time had a great interest in general social education, and did everything to propagate scientific knowledge among the masses. Mystic romanticism, it seems to me, is trying to fill the vacancy. Standing against the isolation of overspecialized sciences, romanticism aims at concretely inspiring the social soul. It is a sort of humanistic movement. But I doubt whether this romanticist activity is able to do what was done by Helmholtz, Huxley, and other scientists.

One more point I wish to notice; and that is, why mysticism came to be so widely welcomed by the public. The progress of science towards the end of the nineteenth century has been so amazingly great that people have been induced to put too much confidence in its possibilities. But as a matter of fact there are a great many problems still left unsolved by science. This opened the eyes of some scholars to the limitations of scientific knowledge, while others have failed to unravel such "miraculous" phenomena as spiritism, telepathy, etc. Tired of fruitless researches and disappointed at unfulfilled expectations, scholars as well as the general public began to look for something else to satisfy their inner yearnings. And they happened to think of the long-forgotten mysticism.

After all, what is necessary at the present juncture, is to start *a humanistic movement founded upon scientific ideas,* to effect a coördination among isolated departmental sciences, and to establish an organic relation between actual life and science.

From the psychological point of view, science belongs to the domain of intellect, and character means individual subjective life. Now this being the distinction between character and intellect, in what relation does one stand to the other? What are the fundamental constituents of character? Can character be moulded independently of the development of intellect?

Among the many constituents of which character is made up, the intellect takes a very prominent place. The widening of a man's intellectual horizon is certainly greatly conducive to the refinement and consolidation of his character. If, on the other hand, he acts hastily, from the lack of sufficient knowledge, as dictated by an impulse of the moment, his character must be said to be standing on a very unsteady basis. Knowledge, especially well-systematized scientific knowledge, is surely needed for the building up of character.

Generally speaking, we can distinguish two forms of knowledge: scientific knowledge and religio-philosophical knowledge. By the former we mean the knowledge that relates to a specified group of facts. Its practical result is the quickening of will-activity rather than the refining influence; it promotes the practicality and definite-

ness of conduct rather than its ennoblement. Religio-philosophical activity, on the other hand, is a synthetic factor. It watches over the totality of things, it grasps the individual's relation to his entire environment. It has, therefore, to be distinguished from the faculty that deals with special laws or a particular mode of activity. Religio-philosophical knowledge, therefore, elevates character to a higher plane of activity rather than making it exact and definite, and secures will-activity on a steady foundation.

These two forms of knowledge, one of which can properly be called scientific and the other religio-philosophical, thus perform two different functions in the formation of character. At this stage of the national development of Japan, which of these kinds of knowledge is more needed, which is more to be developed, than the other? It is of course difficult to form a definite opinion about these things; but to my mind, one is needed just as much as the other, and both of them are still in an adolescent stage of development. While we have to do all we can to further the dissemination of scientific knowledge, we must not neglect to encourage the cultivation of religio-philosophical knowledge.

The fact that scientific knowledge can be acquired from without, but religio-philosophical knowledge must be evolved from within, is readily explained. The former can be obtained without great difficulty by any fairly-endowed mind either through instruction or through self-application. To develop the latter, however, it is necessary to have a strong, comprehensive will, fine sentiment, steady aspirations, keen insight into life, and various inner experiences. Moral discipline, therefore, is indispensable here. As was once discussed by Socrates and Plato in ancient times, morality is not a thing which can be grafted into one's being from without. Unlike scientific knowledge, it must be created by each individual from within. It is truly said that every man is his own philosopher. However wide or however narrow a man's sphere of interest in life, he must have his own philosophy that comprehends and is in harmony with his entire environment. It is the same with a petty tradesman whose store front does not exceed more than a few yards, as with a great statesman on whose shoulders rests the fate of em-

pires. Every man must have a philosophical knowledge comprehensive enough to grasp the whole situation to which he has to adjust himself. With a successful business man every dollar and cent must be accounted for, for otherwise he will sooner or later fail. Therefore, philosophical knowledge has to do with the entirety of things, whatever their importance and consequence. Its characteristic is completeness and universality. To the development of character, this form of knowledge is highly essential, and it would be truer to say that every man should be a philosopher than to say that every man is a philosopher.

Some people imagine that philosophy is a dry, tasteless, and uninteresting affair, having no direct important bearing on practical life. To illustrate that this popular notion is not justified, let me say a few words about love. This sentiment is no doubt at first awakened by some organic impulses, but gradually losing its original significance, it becomes spiritualized. For this spiritualization two things are necessary. One is esthetical association and the other generalization. Sexual love may start with an individual admiration of beauty in the other sex, but as the soul expands and elevates itself, the sentiment of love grows beyond the pale of individualism and begins to cover a wider field, finally embracing entire humanity under its wings. This love, now purely spiritualized, is Platonic. It is then evident that the two things, association and generalization, are necessary to enoble and spiritualize a sentiment which was primarily confined to some particular object.

Some philosophers, especially those of the Middle Ages in Europe, were wont to take into consideration only the element of generalization in philosophical knowledge. This naturally led the common people to understand by philosophy anything but what is interesting and inspiring, while in point of fact philosophy does not exist independent of human sentiment. But the sentiment becomes widened through the generalizing process of philosophical speculation, while it retains its original character of concreteness throughout. Here we have the conception of a universal concrete. While still maintaining its realistic effect, the sentiment moves in unison with general concepts. To illustrate this, take a dramatic work

or a novel. It is an artistic delineation of concrete facts such as we observe in our daily life and is not the presentation of an abstract theory such as we have in Lao-Tze's Tao Teh King or Chwang-Tze's philosophy. Yet we have in the concrete statement of a novel or a dramatic production some general truth reflected or suggested. Every particular incident as it is depicted awakens our associations with things general and conceptual.

This universal concreteness of a sentiment is essential in the development of character. People who act only according to impulses of the moment may be innocent enough, but cannot be said to have any character. The worth of a man's character is betrayed, however, when his apparently meaningless acts, such as laughing, smiling, or chatting, reflect some general concepts deeply ingrained in his being. Sometimes, students disciplining themselves in the Dhyana practice of Buddhism or in the Taoistic doctrine of Lao-Tze and Chwang-Tze, may wrongfully interpret the significance of their master's teachings. They begin to think that the most essential qualification of a philosopher is absolute transcendentalism or utter indifference to worldly interests. The truth is, we cannot escape this earthly entanglement, we must laugh or weep as every other fellow-being, but we can make our every particular act meaningful and reflect in it some grand universal truths. This philosophical discipline is essential to culture.

Next comes practical discipline, which means the adjustment of thought to action. It frequently happens in our daily life that we are unable to execute what we know we ought to do. It is not that we are too lazy, but simply that we are not sufficiently disciplined. The channel between central motives and motor nerves has not been cleared sufficiently for an idea when awakened to find its unobstructed course in action.

As the last essential factor of ethical culture, we have to refer to the strength of will, of which so much is spoken lately in this country. What is meant by a strong will? Can the will be strengthened by discipline? Is a strong character born and not made? These are the questions I wish to discuss here. But before doing

so it is necessary to see what constitutes the will from a psychological point of view.

There are among modern psychologists so many conflicting theories about the nature of the will, that they have not yet come to any definite settlement, but this is not the place to enter into a discussion and criticism of those various will-conceptions. Let me state my own view here and apply it to the practical subject under consideration.

Several conditions necessary to make up will-activity are: (1) End-concept; (2) Motive; (3) Adaptation of the peripheral organs; (4) Persistency of the motive, which comes from the conjunction of end-concept and motive; etc. The first is a simple concept; the second is a feeling of uneasiness; the third is the physical action such as talking, gesturing, or any other mode of adjusting the muscles to the central motive; the fourth is a continued state of uneasiness. When the conjunction of the end-concept and the motive is effected, the latter will persist in the satisfactory adjustment of the peripheral organs, so long as the feeling of uneasiness is not eliminated. A thirsty person will not be quieted until he secures a copious drink of water. If, however, the adjustment of the peripheral organs is not up to the expectation and fails to satisfy the inner needs, there are only two ways between which the individual must choose. The first is the unyielding persistence of a dissatisfied motive which insists upon repeatedly trying the adjustment of the organs, until the result is entirely gratifying to it. The second way is the disappearance of the dissatisfied motive itself. Not being able to overcome the difficulties besetting the final adjustment, the motive yields to them in consequence of its weakness.

For instance, I sometimes take a fancy to play at chess and try to learn the game. But after a few vain attempts I soon come to the conclusion that it is not worth my while, and then abandon it altogether. For in this case my motive is not backed by any strong desire arising from higher aspirations. But it is not so with those experts who, spurred by some irresistible impulses, never relax their efforts to master the art.

Speaking of matters of more serious importance, might there

be any artificial method by which the lack of a strong inner impulse can be remedied? In my opinion, this is done by the cultivation of the foundation of will which consists in a harmonious blending of natural inclination and religio-philosophical knowledge. Ethics, which is still in its primary stage, has not yet given any definite name to what I call here the foundation of will. Some call it will-motive and others end-concept. But neither term is quite accurate, for in our active life it cannot be so analyzed. It may appear to be one or the other according to our viewpoint, but its real significance lies not in the analysis, but in the synthesis in which end-concept and motive-feeling work as one. This synthesis cannot properly be called the will, for it is the source from which the will derives its vitality. Scholars have not named this definitely. I sometimes term it the "foundation of the will," but may it more properly be called the "nucleus of personality"? It is well known in biology that the vitality of a cell depends upon its nucleus. When a protozoic cell is cut in two, the nucleus being left entire in one half, this half continues to grow and heals its wound completely; but the other half in which there is no nucleus, dies. In a similar fashion, the nucleus of personality is, on the one hand, an end-concept of spiritual activities of a man, and, on the other hand, their ground-motive constituting the vital principle of his personality.

Now, that which is most essential in building up a strong, noble chaarcter is to nourish this nucleus of personality. When a man loses his hold on it, he must be said to be spiritually dead. Nobody is perfect, and we are all liable to err frequently, but these errors are not to be censured very severely as long as a man is in possession of a strong, healthy, growing nucleus of personality, for he is sure to recover from the wounds temporarily inflicted and will grow perhaps with more energy. As was seen before, the nucleus must have as its constituent element a comprehensive concept such as Mencius's Vast Energy (*Hao jan chih ch'i*), Christian God, Buddhist Amitabha, or ethicist's humanity. And at the same time it must have this universal concept organically coupled with a healthy, pure sentiment, such as Confucian fellow-feeling (*jên*), Christian love, Buddhist mercy (*karuna*), or ethicist's philanthropy.

The man who has these two essentials coöperating in the make-up of his nucleus of personality, is truly to be called great.

The final issue of our spiritual culture turns on the development of this moral nucleus. To a certain degree, it is a matter of predisposition or heredity. But by accelerating religio-philosophical activities in our soul-life, and by having this soul-life coupled with the noblest sentment, and finally by disciplining oneself in its actualization, much can be achieved by a fairly-endowed soul.

To conclude, it is science that furnishes us with the necessary knowledge of dealing with life and things, and makes our conduct accurate and definite and practical; and in this respect science does much toward moral culture. We cannot do very well without it; science and culture must supplement each other. But as to the establishment of the central nucleus of personality on a solid foundation, we must look for religio-philosophical activities which are vigorous and unbiased, while the scientific knowledge which concerns itself with our natural wants and activities may be compared to the protoplasmic matter that surrounds and feeds the nucleus.

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TOKIO, JAPAN.

THE CONCEPTION OF THE SOUL AND THE BELIEF IN RESURRECTION AMONG THE EGYPTIANS.

WHEN students of comparative religion began to collect their data of the several faiths of primitive peoples, they were astonished to find that a belief in the immortality of the soul was all but universal; and there are many scholars who look upon this unanimity as a proof that the idea is inborn in man and that this *consensus gentium,* so called, is a strong argument in favor of its truth.

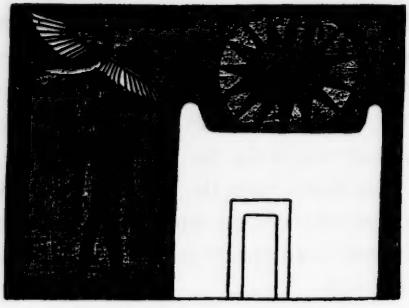
There is a good reason for the prevalence of the belief in immortality, and it is based upon the fact that primitive peoples do not discriminate between dreams and reality. Dreams are real to them, and so if their dead appear to them in dreams, they believe them to be living still.

We must, however, modify our statement, lest we be misunderstood. To say "Savages *believe* in immortality," is, closely considered, wrong. It is not a "belief" with them, not a religious doctrine, nor even a conviction of any moral tenor. To them it is simply a fact of immediate experience.

Savage psychology has been studied in several parts of the globe, and the similarity of its essential features among the different tribes of all continents is remarkable. Indeed, if we consider the logic of primitive man in face of the facts which confront him, we have to understand that to his unsophisticated mind the dead are actually present when they appear in dreams. Savages do not philosophize on the subject, nor do they formulate a *credo.* They see the dead in dreams and visions; they hear their voices; they

converse with them; they consult with them. To question their existence would be as ridiculous to them as to doubt their own being or the actuality of material bodies. What to later generations changes to belief is to them knowledge. Doubt is a creation of incipient civilization, when ideas begin to be "sicklied o'er with the pale cast of thought."

We know of no time when the Egyptians did not believe in immortality, and we may assume that the aborigines as well as the Punt invaders had both some notions of the fate of the soul after death. Their ideas must have been hazy, for in different districts different notions seem to have prevailed, many of which survive in later historical documents. The result is that while all the Egyptians may fairly well be said to have believed in an immortality of



THE KHAIBIT AND THE BA.*

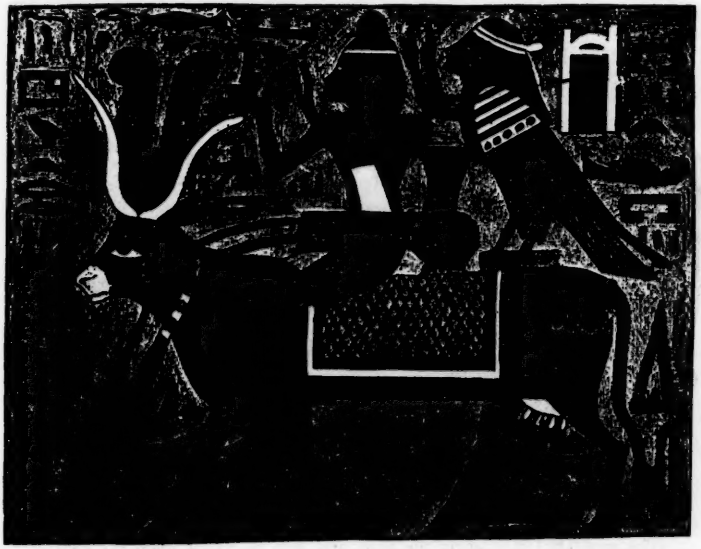
some kind, there are different views, and it is obvious that they have never been systematized into one consistent formula of the Egyptian faith.

We may enumerate many different conceptions of souls, all of which play an important part in the Egyptian religion, and yet we are not informed whether the Egyptians believed in all of them at once, or whether some of them are different names for the same or approximately the same thing, or finally whether we have sometimes to deal with heretical opinions.

The probability is that in some districts the soul was regarded

* From Naville, *Das Thebanische Todtenbuch*, I, plate CIV. — Maspero, *Dawn of Civilization*, p. 108.

as a shadow image, an unsubstantial and idealized shape of the body; in others it was thought to be a bird, a hawk or a phoenix. Later on, it became a human-headed hawk, a mysterious being with wings. Again, it was regarded as a spiritual essence, man's energy and will-power, obviously the product of philosophical reflection. Those who had a vague idea of the significance of the heart-beat looked upon the heart as the seat of the soul, and hence, as the organ of consciousness. All in all, we have no less than nine con-



THE DEAD MAN AND HIS SOUL, THE BA, ON THE GOOD COW HATHOR.*

ceptions of the soul, which occur side by side in the same papyri of the great books of Egypt, among which the Book of the Dead is the most important one.

This mysterious work, the Book of the Dead, is a collection of prayers or incantations, which the soul must recite on its journey to the other world. The name has been given it by modern Egyptologists, because the several chapters of it have been discovered

* From Leemans, *Monuments Egyptiens*, I, III, pl. XII; Maspero, *Dawn of Civilization*, p. 187.

in sarcophagi wrapt up with the dead, but the title is, to say the least, highly inappropriate. The best Egyptian name for it would be, as Budge proposes to translate it, "The Coming Forth by Day," meaning thereby that the soul, in its passage through the underworld, will rise again with renewed life, as the sun, after having set in the West, comes forth again in all his glory in the East. In brief, these prayers are intended for the protection of the soul, and if we had to modernize the name, we ought to call it the "Book of Resurrection."

* * *

The prehistoric inhabitants of Egypt buried their dead in crouched positions with knees drawn up to their faces, on mats

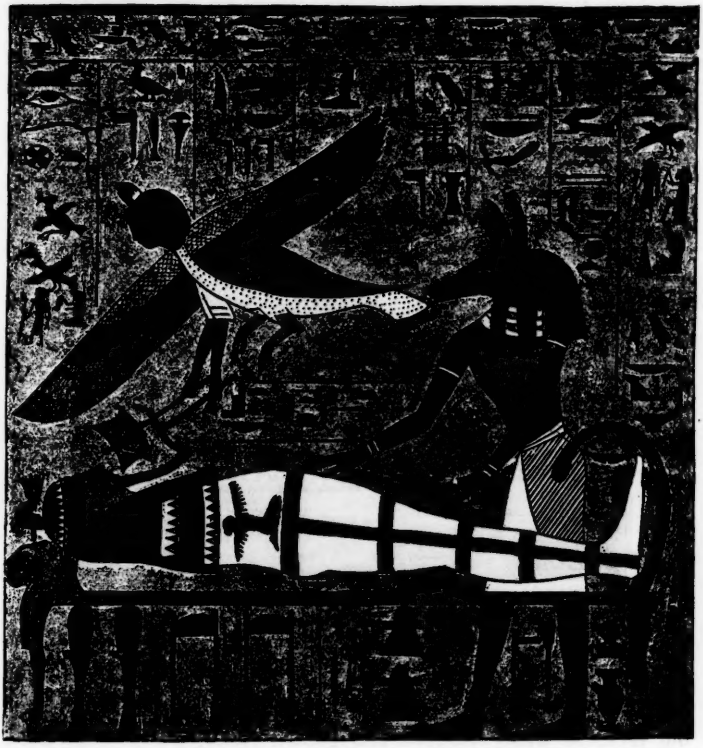
THE SARCOPHAGUS OF THE SCRIBE RA.*
The soul (*ba*) is visiting the body and lays its hands upon the heart of the mummy.

or gazelle skins. There are instances in which the bodies were mutilated, with heads severed from the trunks, and in some cases there are indications of a religious cannibalism. This means that parts of the flesh had been removed for the purpose of being eaten, presumably by the heirs in order to symbolize the transference of the soul of the deceased upon his descendants.

The historical Egyptians, who may have been an entirely new

* From a photograph by Emil Brugsch-Bey. Maspero, *Dawn of Civilization*, p. 199.

race (probably a mixture of the descendants of the Punt invaders with the aborigines), developed a definite system of preserving the bodies by embalming. The reason for this practice must have lain in the belief that the fate of the soul after death depended mainly



OSIRIS ON THE FUNERAL COUCH.*

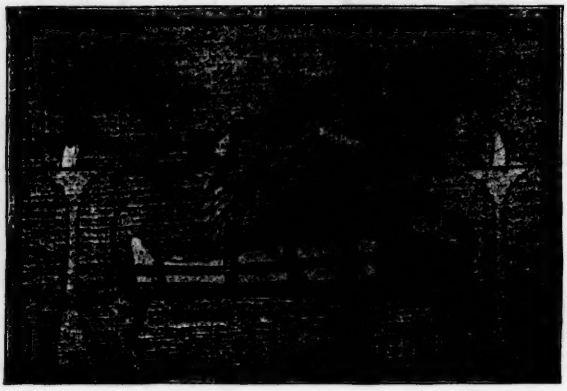
While Anubis prepares the mummy, the soul holds to its nostrils the scepter and the wind-filled sail, the former symbolizing the power of renewal, the latter the breath of life.

on the preservation of the body, and the idea of the significance of the body in connection with the belief in a resurrection has been pre-

* From Rossellini, *Monumenti Civili*, pl. CXXIV, 2; Maspero, *Dawn of Civilization*, p. 179.

served through the history of Egypt. Indeed it has survived in the Christian doctrine of the resurrection of the flesh and is still incorporated in the confession of faith which is recited each Sunday from every orthodox altar, Protestant as well as Greek and Roman Catholic, although actual belief in it is rapidly fading from progressive Christendom.

The ideals that underly the Christian conception of Christ the Saviour, are not foreign to the ancient Egyptians, but the part which Christ plays in Christianity is there ascribed to several deities. First of all he is typified in the god Osiris who, too, had to die after



THE SOUL VISITING THE MUMMY.
(From the Papyrus of Ani.)

he had lived on earth as a man and like unto any other man. Having died, he was embalmed and guided through the under-world by his brother-god Anubis. Anubis accordingly played in this respect the part of Christ not only for Osiris, but for all the dead of Egypt, and we know that among the early Christian Copts, Christ was frequently identified with Anubis. He was the Greek Hermes, who is called by Homer *psychopompos,* leader of souls.

The idea of the soul visiting the body was dear to the Egyptian, for it is represented again and again in papyri, on wall frescoes, and illuminated sarcophagi. The soul's visit to the mummy must

have meant a temporary resuscitation of the dead person, and it was for his benefit that libations and sacrificial meals were offered at the tombs.

* * *

No doubt the Egyptian soul-conception is typically Egyptian. We do not find it in the same shape anywhere else on the surface of the globe. Nevertheless, it is also typically human, and sums up in a matured and cultured form the soul-conception of savage life as it is known to us in Africa, Oceania, North America, and in stray historical records of the primitive people of Europe and Asia.

In spite of the typical peculiarities which confront us in the Egyptian monuments, Egyptian life and Egyptian religion form a

THE MUMMY AT THE TOMB PROTECTED BY ANUBIS.
(From Wiedemann's *Religion of the Ancient Egyptians.*)

definite phase characteristic of a certain stage in the development of mankind. The ingredients which do not go back to the aboriginal inhabitants are partly Lybian, partly Abyssinian, partly Punt Arabic, partly Canaanitish, and a mixture of all these elements with a few incidental ingredients from other countries: Assyria, Phœnicia, and Greece; but the general result is decidedly human, and that is the reason why it still attracts and fascinates us. Moreover, Egyptian views have entered into the life of our present civilization, and in this sense the Egyptians are as much our spiritual ancestors as are the Greeks and the Israelites.

Professor Budge in his recent work on *The Gods of the Egyp-*

tians, characterizes the situation in the Preface (pp. xiv-xvi) as follows:

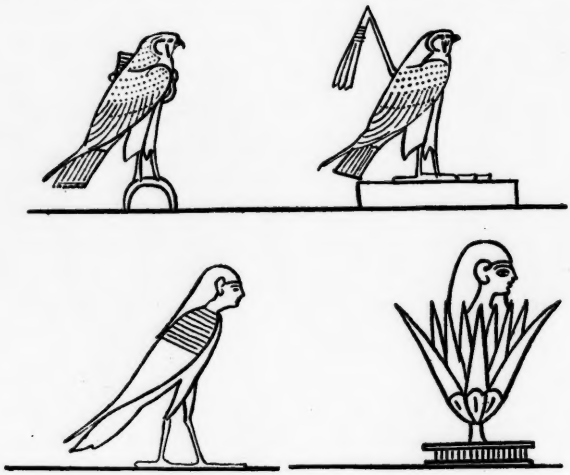
"The cult of Osiris, the dead man deified, and the earliest forms of his worship, were, no doubt, wholly of African origin; these are certainly the oldest elements in the religion of the Dynastic Period, and the most persistent, for Osiris maintained his position as the god and judge of the dead from the Predynastic to the Ptolomaic Period. The Followers of Horus, who brought a solar religion with them into Egypt from the East, never succeeded in dislodging Osiris from his exalted position, and his cult survived undiminished notwithstanding the powerful influence which the priests of Rā, and the worshipers of Amen, and the votaries of Aten respectively exercised throughout the country. The heaven of Osiris was believed to exist in a place where the fields were fertile and well stocked with cattle, and where meat and drink were abundant; the abodes of the blessed were thought to be constructed after the model of the comfortable Egyptian homesteads in which they had lived during life, and the ordinary Egyptian hoped to live in one of these with his wives and parents. On the other hand, the followers of Rā, the sun-god, believed in a heaven of a more spiritual character, and their great hope was to occupy a seat in the boat of the god, and, arrayed in light, to travel whithersoever he went. They wished to become bright and shining spirits, and to live upon the celestial meat and drink upon which he lived; as he was so they hoped to be in every respect. The materialistic heaven of Osiris appealed to the masses of Egypt, and the heaven where Rā lived to the priests of Rā and other solar gods, and to royal and aristocratic families, and to the members of the foreign section of the community who were of Eastern origin.

The various waves of religious thought and feeling, which swept over Egypt during the five thousand years of her history which are known to us, did not seriously disturb the cult of Osiris, for it held out to the people hopes of resurrection and immortality of a character which no other form of religion could give. Secure in these hopes the people regarded the various changes and developments of religious ideas in their country with equanimity and modifications in the public worship of the gods, provided that the religious fasts and processions were not interrupted, moved them but little. Kings and priests from time to time made attempts to absorb the cult of Osiris into religious systems of a solar character, but they failed, and Osiris, the man-god, always triumphed, and at the last, when his cult disappeared before the religion of the Man Christ, the Egyptians who embraced Christianity found that the moral system of the old cult and that of the new religion were so similar, and the promises of resurrection and immortality in each so much alike, that they transferred their allegiance from Osiris to Jesus of Nazareth without

ANUBIS USHERING THE DEAD INTO THE PRESENCE OF OSIRIS.
(After a colored facsimile of a picture in the *Book of the Dead*, by Pleyte.)

difficulty. Moreover, Isis and the child Horus were straightway identified with Mary the Virgin and her Son, and in the apocryphal literature of the first centuries which followed the evangelization of Egypt, several of the legends about Isis and her sorrowful wanderings were made to center round the Mother of Christ. Certain of the attributes of the sister goddesses of Isis were also ascribed to her, and, like the Goddess Neith of Saïs, she was declared to possess perpetual virginity. Certain of the Egyptian Christian Fathers gave to the Virgin the title "Theotokos," or "Mother of God," forgetting, apparently, that it was an exact translation of *neter mut*, a very old and common title of Isis."

The body of man was called *khat*[1], and was represented in hieroglyphics by a dead fish and a perfume bottle, indicating in



FOUR REPRESENTATIONS OF THE SOUL.
(From Lenormant's *Histoire de l'Orient*, III, 269.)

their combination putrid odor. It was also written in a fuller form,[2] which means something subject to decay that can be preserved by mummification. The hope of the Egyptians for immortality being closely affiliated to the idea of the restitution of the body, they were bent on preserving its form, which gradually led to the practice of mummification.

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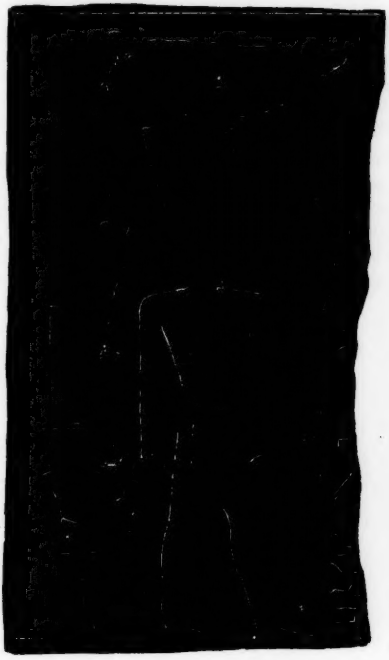
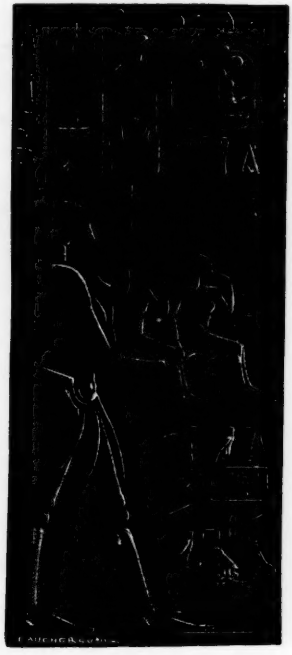
[1]

[2]

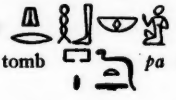
The tomb was built to be the residence of the mummy for all time to come, and was hence called "the eternal house," *pa t'etta;* and we must assume that there, at appointed seasons, comparable to our All Souls' Day, memorial services were held with libations, food-offering, and incense-burning.[3]



KING THUTMOSIS II ACCOMPANIED BY HIS DOUBLE.*



A ROYAL INFANT AND HIS DOUBLE.†

[3] The priest is called *ker heb*  which signifies one who conducts the festivals at the tomb *pa t'etta,* the everlasting house.

* The king belongs to the XVIIIth dynasty; his double carries on his head the king's *ka*-name. (From Arundale-Bonomi-Birch, *Gallery of Antiquities from the Br. M.*, pl. 31. Maspero, *Dawn of Civilization,* p. 261.)

† This bas-relief in the temple of Luxor represents the birth of Amenothes III. From a photograph by Gayet; Maspero, *Dawn of Civilization,* 259.

The soul is represented in many ways, either as a bird,[4] or as a hawk,[5] or, most commonly, as a human-headed hawk,[6] called *ba*.

The *ba* represents mainly the functions of consciousness and is supposed to visit the tomb from time to time, and enter into the *khat*, the perishable body. In fact, the purpose of the *khat's* mummification is simply to make it possible for the soul to enter again into its body.

Another conception of the soul is the idealized shadow of a man, called "the shade," which in hieroglyphics is called *khaibit*.[7]

A typical Egyptian view of the soul is a description of the sentiment that throbs in our breast—that part of the body that lies between the arms and finds a vivid expression in the use of our hands. It is called *ka* and is pictured in hieroglyphics by two outstretched arms,[8] which is commonly translated "double," for it is supposed to be the ethereal shape of the man and represents the personality as a kind of astral body, which is supposed to be in possession of all attributes of the man to whom it belongs. The translation "double" is in so far justified as the monuments actually represent the *ka* as a second and an additional figure, which, at certain times and certain places, is deemed necessary to add to the representation of a man. We see, for instance, the picture of a new-born prince in which his double, his idealized self, is represented right behind him, bearing a special name, the so-called *ka*-name of the future king.[9]

The conscience of the man, the organ of his moral life, is supposed to have its seat in the heart, hence *ab*, the heart, is the name of the soul in a similar sense as even to-day we would use the word heart. It is written in hieroglyphics in two ways.[10]

The spirit of a man is called *khu*, represented as an ibis,[11] the emblem of Thoth, the scribe of the gods, the mediator between

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[4] [5] [6] [7] [8] Coptic

[9] The *ka*-name is indicated by resting on the hieroglyph *ka* and having on top the hawk of Hor.

[10] or [11]

man and the celestials, the protector of science and the divine incarnation of the Word, the Logos—a conception which plays an important part in Egyptian theology.

Another way of representing the soul is as the vital force, called *sekhem*, represented in hieroglyphics by a symbol that seems to be a fan, representing breath, vitality, and energy.[12]

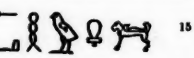
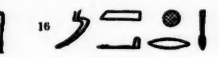
Finally, the personality of man is covered by all that appertains to his name, and thus it is represented in Egyptian by the hieroglyph *ren*,[13] which means "name."

The body when mummified is called *sahu*[14] and is pictured as

RECEIVING THE BREATH OF LIFE.*

THE DEAD MAN AND HIS SOUL.†

an upright mummy.[15] When the deceased has been justified before the judgment throne of Osiris, and when his heart has been returned to him, he is regarded as having received the benediction of truth and becomes *maa-kheru*,[16] a word which finds an equivalent in the German *selig*, and is translated in English, "triumphant,"

12 13 14  15 16 

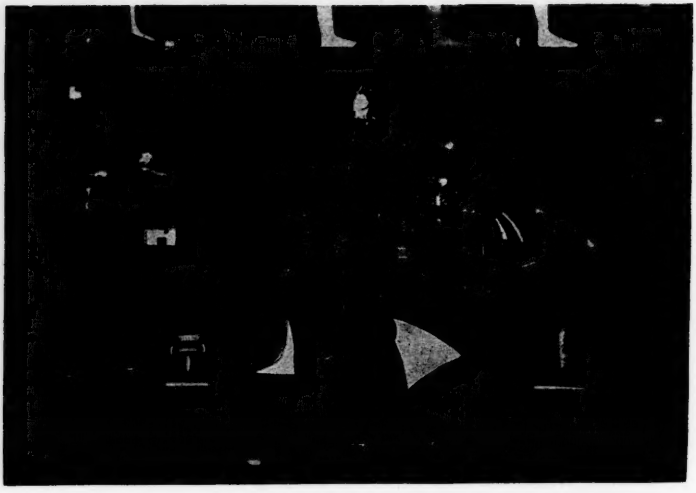
* From Naville, *Das Aegyptische Todtenbuch*, I, pl. LXIX. Maspero, *Dawn of Civilization*, p. 217.

† From Guieyesse-Lefébure, *Le Papyrus de Soutimès*, pl. VII.—Maspero, *Dawn of Civilization*, p. 183.

"justified," "victorious," or "sainted." When the body has been sainted, it is supposed to be in possession of a spiritual body; it becomes luminous and is possessed of an incorruptible *sahu*, a transfigured body.

Man's resurrection soul is characterized by the bird *bennu*,[17] the Egyptian phœnix.

The idea of resurrection has always been the main doctrine of the religious life of Egypt. Here all longings find their satis-



WEIGHING THE HEART.

Anubis adjusts the tongue of the balance the construction of which is noteworthy. A feather, the emblem of truth, serves him for a weight. (From the Papyrus of Ani.)

faction, here all interests converge, and here all hopes are centered. When a mummy is removed in a boat to its eternal resting-place, a near relation of the deceased stands in the bow of the boat and calls to the helmsman:

"Steer to the West, to the land of the justified.
The women of the boat weep much, very much.

[17]

In peace, in peace to the West, thou blessed one, go in peace!

. . . . . .

When time has become eternity then shall we see thee again.
For, behold, thou goest away to that country in which all are equal."[18]

* * *

All the amulets which were worn by the living or were placed upon the mummy to accompany the dead to the other world, are intended to serve the purpose of insuring a happy resurrection on the day when time will become eternity.

The most common symbols used are the *ankh*,[19] called also the "key of life," or *crux ansata* (the handle cross), or the Egyptian cross. It means "life" and is seen in the hands of the gods as an emblem of their divinity.

Another symbol is the *tet* or *ded*,[20] the backbone of Osiris, a symbol of stability.

A third symbol is the scepter *usr*,[21] meaning "strength," having on top a hook not unlike the head of the oryx (an animal sacred to Set) and ending below in a horse-shoe form.

Still another symbol is the feather[22] of truth worn by the goddess Maat on her head. It means "purity," "faithfulness," and "justification."

The vulture,[23] representing "Mother Isis," was placed on the neck of the mummy on the day of the funeral.

The *uræus* (snake),[24] like the vulture, is a symbol of Isis, the two being sometimes combined. The former represents Upper Egypt and is frequently painted with outstretched wings as hovering over the king; the latter received particular veneration in the Delta. Both were also worshiped as special goddesses, the vulture under the name Nekhbit, the *uræus* (snake) under the name Uazit.

The buckle or tie, called *thet*,[25] is one of the commonest amulets found in the graves. It is commonly made of red jasper, cornelian, porphery, red glass, red faience, or sycamore wood; and we are

[18] Ermann, *Life in Ancient Egypt*, chapter on "The Dead," pp. 320-321.

[19] [20] [21] [22] [23] [24] [25]

told that the red color represents the blood of Isis. It is placed on the neck of the dead.

The symbol *nefer*,[26] originally representing the heart with the trachea, but later on interpreted as a lute, means beauty, gladness, joy, and good luck. It is frequently trebled so as to mean "thrice blest."

The symbolical eye, *utat*, made of glazed faience, wood, granite, haematite, cornelian, lapis lazuli, or precious metals, is shaped either as the right[27] or the left[28] eye or both in combination. Sometimes the right eye is called the sun and the left eye the moon; and in other passages the former is explained as the eye of Hor in the south, meaning the sun in day-time, and the left eye, the eye of Hor in the north, meaning the moon during the night. The eyes of Hor are endangered by Set but are known to be always victorious. Frequently they bear the inscription *uza*, i.e., "prosperous" or "hale," and the souls of the dead were believed to be safe under their protection.

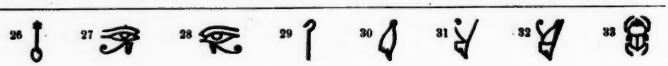
In the Book of the Dead, the *utats*, the eyes of Hor, are painted with wings and human legs.

The crook *hek*[29] signifies the care that the gods take of mortals, and its use continues in the Christian Church as an emblem of episcopal responsibility.

Other symbols representing royalty are the white crown of the south, *het*,[30] the red crown of the north, *tesher*,[31] and the double crown[32] of both Upper and Lower Egypt, called in later times *pschent*.

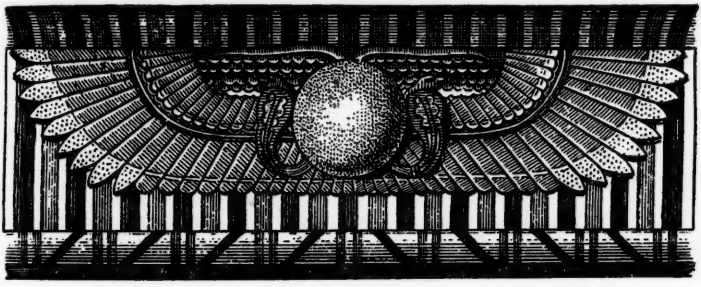
The scarab, *kheper*,[33] the Egyptian dung-beetle (*ateuchus sacer*) was considered with special awe, and it meant generation or regeneration, transformation, resurrection, self-creative power, and immortal life.

The Egyptians had observed the scarab roll a little mud ball and hide it in a safe place. In due time the young beetles came out of this mysterious ball, and it was assumed that the scarab had no sex, but that it possessed the power of regenerating itself.



The heart, *ab*,[34] is also considered as an important amulet, and Mr. Budge quotes one instance in which a heart amulet bears, on one side, the inscription of the name of the goddess Neit, a picture of the bird *Bennu*, and the legend *Nuk ba Khepara*, that is, "I am the soul of life eternal"; and, on the other side, the chapter on "The Heart" quoted from the Book of the Dead.

We must mention also the symbol *hefnu*, which means "a myriad" and is represented as a frog,[35], being the emblem of the goddess Hekt, a form of Hathor, wife of the god Khnemu. Hekt also was believed to have a favorable influence upon man's resurrection. Even as late as in the Hellenistic period, and still in the Roman period of Egyptian history, we find frogs on lamps, and in



THE SUN OF RIGHTEOUSNESS WITH HEALING IN HIS WINGS.

one instance the frog bears the inscription in Greek, "I am the resurrection."[36]

An enumeration of Egyptian symbols would not be complete without finally mentioning the emblem of the winged disk, which appears over every temple entrance in Egypt. There is a legend about its introduction, which relates that Râ Harmakhis, "the Ever-living Sun-god," was confronted with the enemies of the gods of the Egyptians, and his son, Hor Behudti, "Hor as a sparrow hawk," struck terror among the host of Set, by assuming the overawing form of a winged disk.

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[34] [35] [36] ΕΓΩ ΕΙΜΙ ΑΝΑCΤΑCΙC.

Thoth, the scribe of the gods, says:

"'The darter of rays who came forth from Râ, he conquered the enemies in his form [of a winged sun-disk]; from this day he shall be called the Darter of Rays who emergeth from the horizon.'

"Hamarkhis spake unto Thoth:

"'Set this sun at every place at which I tarry, at the places of the gods in the South Land, at the places of the gods in the North Land, [at the places of the gods] in the Underworld, that it may banish evil from their vicinity.'

"Thoth set this form at every spot, at every place, how many soever there were, at which any gods or goddesses might be. And this is the winged sun-disk which is over the sanctuaries of all the gods and goddesses in Egypt, for their sanctuary is also that of Horbehûdti."[37]

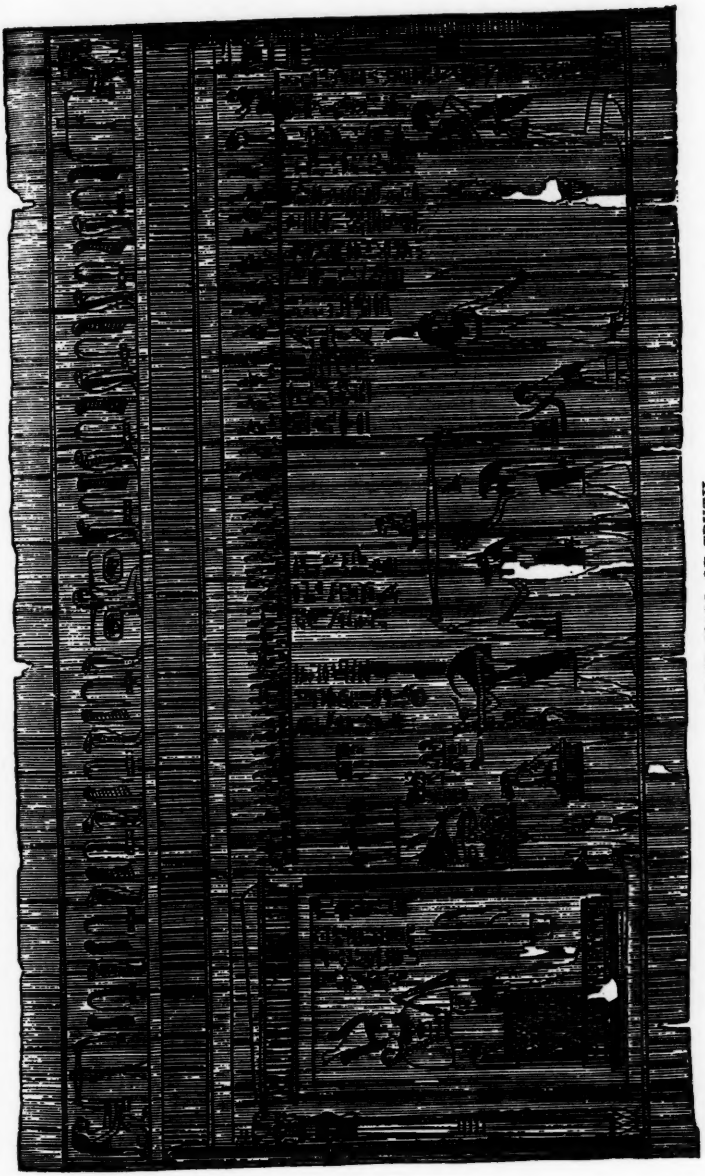
The winged disk, accordingly, as related in this legend, "banishes evil from the temples." It is the emblem of rescue from enemies and of salvation. The same emblem is used in other Oriental countries, in Arabia, Phœnicia, Syria, and especially in Babylonia, and we must assume that even the Israelites had no objection to it. At any rate, we find an allusion to it in the prophet Malachi (iv. 2), who apparently refers to this emblem of the deity, when he speaks of Yahveh as the "Sun of righteousness....with healing in his wings."

* * *

A prominent feature was the effect which the belief in immortality had on Egyptian morals. The soul could pass easily in its migrations through the shadows of the under-world if it had not committed any offense against either the gods or its fellow beings. It had to know the magic spells that were required to overcome the powers of darkness, and when finally it reached the hall of truth, the heart of the deceased was weighed in the balance with truth, which is represented pictorially by a feather.

The deceased makes a negative confession to forty-two judges of the sins which he has abstained from committing, and we quote

[37] Alfred Wiedemann, *Religion of the Ancient Egyptians*, p. 74.



THE HALL OF TRUTH.

the following examples from the Papyrus of Nu (Budge, *Egyptian Ideas of the Future Life*, pp. 130-134):

"I have not done iniquity.—I have not committed theft.—I have not made light the bushel.—I have not acted deceitfully.—I have not uttered falsehood.—I have not uttered vile words.—I have not eaten my heart (i. e. lost my temper and become angry).—I have not pried into matters to make mischief.—I have not set my mouth in motion against any man.—I have not polluted myself.—I have not made any man to be afraid.—I have not made myself deaf unto the words of right and truth.—I have not made another person to weep.—I have not behaved with insolence.—I have not increased my wealth except by means of such things as are mine own possessions."

By his justification he becomes identified with Osiris who now lives in him as a power of salvation. We quote from the Papyrus of Ani (*Book of the Dead*, Vol. I, p. 29, Chicago, The Open Court Publishing Co.), where Horus, the son of Isis, the avenger of his father Osiris, and the saviour of mankind, addresses Osiris Un-nefer:

"I have come to thee, O Un-nefer, and I have brought unto thee the Osiris Ani. His heart is [found] righteous, and it hath come forth from the balance; it hath not sinned against any god or any goddess. Thoth hath weighed it according to the decree pronounced unto him by the company of the gods; and it is most true and righteous. Grant that cakes and ale may be given unto him, and let him appear in the presence of the god Osiris; and let him be like unto the followers of Horus for ever and for ever."

Such in main outline are the leading facts in the Egyptian conception of the soul and its life after death, and closely considered they are but the natural outcome of those views which can be observed in all the prehistoric nations of the world; but in the case of Egypt they are reduced to a clear conception, symbolized by appropriate emblems, stated in religious doctrines, systematically applied to practical life in the shape of moral maxims, and pictured graphically in religious art.

EDITOR.

MAGIC SQUARES.

MAGIC squares are of themselves only mathematical curios, but they involve principles whose unfolding should lead the thoughtful mind to a higher conception of the wonderful laws of symphony and order which govern the science of numbers.

The earliest record of a magic square is found in Chinese literature dated about A. D. 1125,* but since then this interesting subject has been more or less studied and developed by mathematicians of all nations.

It is the writer's purpose to present some general and comprehensive methods for constructing magic squares which he believes to be original, and also to briefly review what is commonly known concerning their construction.

THE GENERAL QUALITIES AND CHARACTERISTICS OF MAGIC SQUARES.

A magic square consists of a series of numbers arranged in quadratic form so that the sum of each vertical, horizontal and corner diagonal column is the same amount. These squares can be made with either an odd or an even number of cells, but as odd squares are constructed by methods which differ from those that govern the formation of even squares, the two classes will be considered under separate headings.

ODD MAGIC SQUARES.

In these squares it is not only requisite that the sum of all columns shall be the same amount, but also that the sum of any

* See p. 19 of *Chinese Philosophy* by Dr. Paul Carus.

two numbers that are geometrically equidistant from the center number shall equal twice that number. Unless these conditions are fulfilled, the square cannot be considered perfect.

The square of 3×3 shown in Fig. 1 covers the smallest aggregation of numbers that is capable of magic square arrangement, and it is also the only possible arrangement of nine different numbers, relatively to each other, which fulfills the required conditions. It will be seen that the sum of each of the three vertical, the three horizontal, and the two corner diagonal columns in this square is 15,. making in all eight columns having that total: also that the sum of any two opposite numbers is 10, which is twice the center number. It is therefore a perfect square of 3×3.

The next largest odd magic square is that of 5×5, and there are a great many different arrangements of twenty-five numbers,

8	1	6
3	5	7
4	9	2

Totals = 15.

Fig 1.

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

Totals = 65.

Fig. 2.

which will show perfect results, each arrangement being the production of a different constructive method. Fig. 2. illustrates what is probably the oldest and best known arrangement of this square.

The sum of each of the five horizontal, the five vertical, and the two corner diagonal columns is 65, and the sum of any two numbers which are geometrically equidistant from the center number is 26, or twice the center number.

In order to intelligently follow the rule used in the construction of this square it may be conceived that its upper and lower edges are bent around backwards, and united to form a horizontal cylinder with the numbers on the outside, the lower line of figures thus coming next in order to the upper line. It may also be conceived that the square is bent around backwards in a direction at right

angles to that which was last considered, so that it forms a vertical cylinder with the extreme right and left hand columns adjacent to each other.

An understanding of this simple conception will assist the student to follow the new methods of building odd magic squares that are to be described, all of these methods being based on a right or left hand diagonal formation.

Referring to Fig. 2, it will be seen that the square is started by writing unity in the center cell of the upper row, the consecutive numbers proceeding diagonally therefrom in a right hand direction. Using the conception of a horizontal cylinder, 2 will be located in the lower row, followed by 3 in the next upper cell to the right. Here the formation of the vertical cylinder being conceived the next upper square will be where 4 is written, then 5; further progress being here blocked by 1 which already occupies the next upper cell in diagonal order.

When a block thus occurs in the regular spacing (which will be at every fifth number in a 5×5 square) the next number must in this case be written in the cell vertically below the one last filled, so that 6 is written in the cell below 5, and the right hand diagonal order is then continued in cells occupied by 7 and 8. Here the horizontal cylinder is imagined, showing the location of 9, then the conception of the vertical cylinder will indicate the location of 10; further regular progression being here once more blocked by 6, so 11 is written under 10 and the diagonal order continued to 15. A mental picture of the combination of vertical and horizontal cylinders will here show that further diagonal progress is blocked by 11, so 16 is written under 15. The vertical cylinder will then indicate the cell in which 17 must be located, and the horizontal cylinder will show the next cell diagonally upwards to the right to be occupied by 18, and so on until the final number 25 is reached and the square completed.

Fig. 3 illustrates the development of a 7×7 square constructed according to the preceding method, and the student is advised to follow the sequence of the numbers to impress the rule on his memory. A variation of the last method is shown in Fig. 4, illustrating

another 7×7 square. In this example 1 is placed in the next cell horizontally to the right of the center cell, and the consecutive numbers proceed diagonally upward therefrom, as before, in a right hand direction until a block occurs. The next number is then written in the second cell horizontally to the right of the last cell filled (instead of the cell below as in previous examples) and the upward diagonal order is resumed until the next block occurs.

30	39	48	1	10	19	28
38	47	7	9	18	27	29
46	6	8	17	26	35	37
5	14	16	25	34	36	45
13	15	24	33	42	44	4
21	23	32	41	43	3	12
22	31	40	49	2	11	20

Fig. 3.

4	29	12	37	20	45	28
35	11	36	19	44	27	3
10	42	18	43	26	2	34
41	17	49	25	1	33	9
16	48	24	7	32	8	40
47	23	6	31	14	39	15
22	5	30	13	38	21	46

Totals = 175.

Fig. 4.

10	18	1	14	22
11	24	7	20	3
17	5	13	21	9
23	6	19	2	15
4	12	25	8	16

Totals = 65.

Fig. 5.

Then two cells to the right again, and regular diagonal order continued, and so on until all the cells are filled.

The preceding examples may be again varied by writing the numbers in left hand instead of right hand diagonal sequence, making use of the same spacing of numbers as before when blocks occur in the regular sequence of construction.

We now come to a series of very interesting methods for building odd magic squares which involve the use of the knight's move in chess, and it is worthy of note that the squares formed by these methods possess curious characteristics in addition to those

previously referred to. To chess-players the knight's move will require no comment, but for those who are not familiar with this game it may be explained as a move of two squares straight forward in any direction and one square to either right or left.

The magic square of 5×5 illustrated in Fig. 5 is started by placing 1 in the center cell of the upper row, and the knight's move employed in its construction will be two cells upward and one cell to the right.

Using the idea of the horizontal cylinder 2 must be written in the second line from the bottom, as shown, and then 3 in the second line from the top. Now conceiving a combination of the horizontal and vertical cylinders, the next move will locate 4 in the extreme lower left hand corner, and then 5 in the middle row. We now find that the next move is blocked by one, so 6 is written below 5, and the knight's moves are then continued, and so on until the last number, 25, is written in the middle cell of the lower line, and the square is thus completed.

In common with the odd magic squares which were previously described, it will be found that in this square the sum of each of the five horizontal, the five perpendicular, and the two corner diagonal columns is 65, also that the sum of any two numbers that are geometrically equidistant from the center is 26, or twice the number in the center cell, thus filling all the general qualifications of a perfect square.

In addition, however, to these characteristics it will be noted that each spiral row of figures around the horizontal and vertical cylinders traced either right handed or left handed also amounts to 65. In the vertical cylinder, there are five right hand, and five left hand spirals, two of which form the two corner diagonal columns across the square, leaving eight new combinations. The same number of combinations will also be found in the horizontal cylinder. Counting therefore five horizontal columns, five vertical columns, two corner diagonal columns, and sixteen right and left hand spiral columns, there will be found in all twenty-eight columns each of which will sum up to 65, whereas in either of the 5×5

squares previously considered there will be found only twelve columns that will amount to that number.

This method of construction is subject to a number of variations. For example, the knight's move may be upwards and to the left hand instead of to the right, or it may be made downwards and either to the right or left hand, and also in other directions. There are in fact eight different ways in which the knight's move may be started from the center cell in the upper line. Six of these moves are indicated by figure 2's in different cells of Fig. 6, and each of these moves if continued in its own direction, varied by regular breaks as before described, will produce a different but perfect square. The remaining two possible knight's moves, indicated by cyphers, will not produce perfect squares.

		1		
0				0
	2		2	
	2		2	
2		.		2

Fig. 6.

		19	2	15	23	
	12	25	8		4	
10	18	1	14	22	10	
11	24	7	20	3		
17	5	13	21	9	17	
23	6	19	2	15		
4	12	25	8	16		

Fig. 7.

It may here be desirable to explain another method for locating numbers in their proper cells which some may prefer to that which involves the conception of the double cylinder. This method consists in constructing parts of auxiliary squares around two or more sides of the main square, and temporarily writing the numbers in the cells of these auxiliary squares when their regular placing carries them outside the limits of the main square. The temporary location of these numbers in the cells of the auxiliary squares will then indicate into which cells of the main square they must be permanently transferred.

Fig. 7 shows a 5×5 main square with parts of three auxiliary

squares, and the main square will be built up in the same way as Fig. 5.

Starting with 1 in the center of the top line, the first knight's move of two cells upwards and one to the right takes 2 across the top margin of the main square into the second cell of the second line from the bottom in one of the auxiliary squares, so 2 must be transferred to the same relative position in the main square. Starting again from 2 in the main square, the next move places 3 within the main square, but 4 goes out of it into the lower left hand corner of an auxiliary square, from which it must be transferred to the same location in the main square, and so on throughout.

The method last described and also the conception of the double cylinders may be considered simply as aids to the beginner. With a little practice the student will be able to select the proper cells in the square as fast as the figures can be written therein.

Having thus explained certain specific and novel lines of construction, the general principles governing the development of all odd magic squares by these methods may now be formulated.

1. The center cell in the square must always contain the middle number of the series of numbers used, i. e., a number which is equal to one half the sum of the first and last numbers of the series.
2. No perfect magic square can therefore be started from its center cell, but it may be started from any cell other than the center one.
3. With certain specific exceptions which will be referred to later on, odd magic squares may be constructed by either right or left hand diagonal sequence, or by a number of so-called knight's moves, varied in all cases by periodical and well defined departures from normal spacing.
4. The directions and dimensions of these departures from normal spacing, or "break moves," as they may be termed, are governed by the relative spacing of cells occupied by the first and last numbers of the series, and may be determined as follows:

RULE: Place the first number of the series in any desired cell (excepting the center one) and the last number of the series in the cell which is geometrically opposite to the cell containing the first number. The relative spacing of these two cells must then be repeated whenever a block occurs in the regular progression.

EXAMPLES.

Using a blank square of 5×5, 1 may be written in the middle cell of the upper line. The geometrically opposite cell to this being the middle cell in the lower line, 25 must be written therein. 1 will therefore be located four cells above in the middle vertical column, or what is the same thing, and easier to follow, one cell below 25. When, therefore, a square of 5×5 is commenced with the first number in the middle cell of the upper line, the break move will *always* be one cell downwards, irrespective of the method of regular

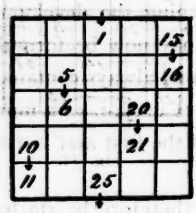


Fig. 8.

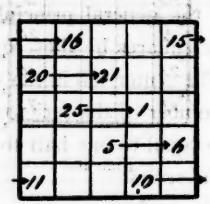


Fig. 9.

advance. Fig. 8 shows the break moves in a 5×5 square as above described using a right hand upward diagonal advance.

Again using a blank 5×5 square, 1 may be written in the cell immediately to the right of the center cell, bringing 25 into the cell to the left of the center cell. The break moves in this case will therefore be two cells to the right of the last cell occupied, irrespective of the method used for regular advance. Fig. 9 illustrates the break moves in the above case, when a right hand upward diagonal advance is used. The positions of these break moves in the square will naturally vary with the method of advance, but the spacing of the moves themselves will remain unchanged.

NOTE: The foregoing break moves were previously described in several specific examples (See Figs. 1, 2, 3, 4, and 5) and the student will now observe how they agree with the general rule.

Once more using a blank square of 5×5, 1 may be written in the upper left hand corner and 25 in the lower right hand corner. 1 will then occupy a position four cells removed from 25 in a left hand upward diagonal, or what is the same thing and easier to follow, the next cell in a right hand downward diagonal. This will therefore be the break move whenever a block occurs in the regular spacing. (See Fig. 10.)

As a final example we will write 1 in the second cell from the left in the upper line of a 5×5 square, which calls for the placing

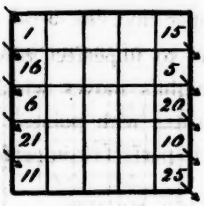


Fig. 10.

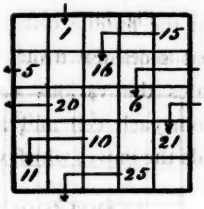


Fig. 11.

of 25 in the second square from the right in the lower line. The place relation between 25 and 1 may then be described by a knight's move of two cells to the left and one cell downwards, and this must be the break move whenever a block occurs in the regular spacing. (See Fig. 11.)

As before stated odd magic squares may be commenced in any cell excepting the center one, and perfect squares may be built up from such commencements by a great variety of regular moves, such as right hand diagonal sequence, upwards or downwards, left hand diagonal sequence upwards or downwards, or a number of knight's moves in various directions. There are four possible moves from each cell in diagonal sequence, and eight possible moves from each cell by the knight's move. The greater number of these moves

will produce perfect magic squares, but there will be found certain exceptions which can be shown most readily by diagrams.

Fig. 12 is a 5×5 square in which the pointed arrow heads indicate the directions of diagonal sequence by which perfect squares may be constructed, while the blunt arrow heads show the directions

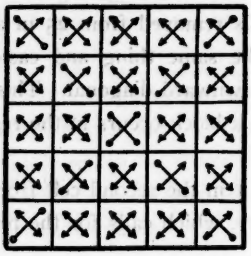


Fig. 12.

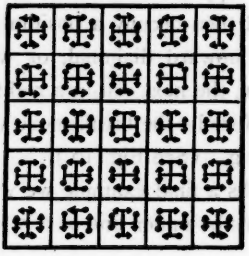


Fig. 13.

of diagonal sequence which will lead to imperfect results. Fig. 13 illustrates the various *normal* knight's moves which may be started from each cell and also indicates with pointed and blunt arrow heads the moves which will lead to perfect or imperfect results.

EXAMPLES OF 5×5 MAGIC SQUARES.

Figs. 14, 15, and 16 show three 5×5 squares, each having 1 in the upper left hand corner cell and 25 in the lower right hand corner cell, and being constructed respectively with a right hand

1	17	8	24	15
16	7	23	14	5
6	22	13	4	20
21	12	3	19	10
11	2	18	9	25

Fig. 14.

1	21	16	11	6
12	7	2	22	17
23	18	13	8	3
9	4	24	19	14
20	15	10	5	25

Fig. 15.

1	11	21	6	16
22	7	17	2	12
18	3	13	23	8
14	24	9	19	4
10	20	5	15	25

Fig. 16.

upward diagonal sequence and right and left hand horizontal knight's moves, the break move being necessarily the same in each example. (See Fig. 10.)

Figs. 17, 18, 19, and 20 show four 5×5 squares, each having 1 in the second cell from the left in the upper line and 25 in the

8	1	24	17	15
5	23	16	14	7
22	20	13	6	4
19	12	10	3	21
11	9	2	25	18

Fig. 17.

15	1	17	8	24
23	14	5	16	7
6	22	13	4	20
19	10	21	12	3
2	18	9	25	11

Fig. 18.

second cell from the right in the lower line, and being built up respectively with right and left hand upward diagonal sequence

22	1	10	14	18
11	20	24	3	7
5	9	13	17	21
19	23	2	6	15
8	12	16	25	4

Fig. 19.

23	1	9	12	20
15	18	21	4	7
2	10	13	16	24
10	22	5	8	11
6	14	17	25	3

Fig. 20.

and upward right and downward left hand knight's moves, and with similar break moves in each example. (See Fig. 11.)

18	10	22	14	1
11	3	20	7	24
9	21	13	5	17
2	19	6	23	15
25	12	4	16	8

Fig. 21.

9	12	20	23	1
18	21	4	7	15
2	10	13	16	24
11	19	22	5	8
25	3	6	14	17

Fig. 22.

12	23	9	20	1
4	15	21	7	18
16	2	13	24	10
8	19	5	11	22
25	6	17	3	14

Fig. 23.

Figs. 21, 22, and 23 illustrate three 5×5 squares, each having 1 in the upper right hand corner and 25 in the lower left hand

corner, and being built up respectively with upward and downward right hand normal knight's moves, and a downward right hand elongated knight's move.

For the sake of simplicity these examples have been shown in 5×5 squares, but the rules will naturally apply to all sizes of odd magic squares by using the appropriate numbers. The explanations have also been given at some length because they cover general and comprehensive methods, a good understanding of which will make the student a master of the entire subject of odd magic squares.

It is clear that no special significance can be attached to the so-called knight's move, *per se*, as applied to the construction of magic squares, it being only one of many methods of regular spacing, all of which will produce equivalent results. For example, the 3×3 square shown in Fig. 1 may be said to be built up by a succession of abbreviated knight's moves of one cell to the right and one cell upwards. Squares illustrated in Figs. 2, 3, and 4 are also constructed by this abbreviated knight's move, but the square illustrated in Fig. 5 is built up by the normal knight's move.

80	58	45	23	1	69	47	34	12
9	65	46	33	11	79	57	44	22
10	78	56	43	21	8	67	54	32
20	7	66	53	31	18	77	55	42
30	17	76	63	41	19	6	65	52
40	27	5	64	51	29	16	75	62
50	26	15	74	61	39	26	4	72
60	38	25	3	71	49	36	14	73
70	48	35	13	81	59	37	24	2

Totals = 369.

Fig. 24.

It is equally easy to construct squares by means of an elongated knight's move, say, four cells to the right and one cell upwards as shown in Fig. 24, or by a move consisting of two cells to the right and two cells downwards, as shown in Fig. 25, the latter being

equivalent to a right hand downward diagonal sequence wherein alternate cells are consecutively filled.

There are in fact almost innumerable combinations of moves by which perfect odd magic squares may be constructed.

39	34	20	15	1	77	72	58	53
49	44	30	25	11	6	73	68	63
59	54	40	35	21	16	2	78	64
69	55	50	45	31	26	12	7	74
79	65	60	46	41	36	22	17	3
8	75	70	56	51	37	32	27	13
18	4	80	66	61	47	42	28	23
19	14	9	76	71	57	52	38	33
29	24	10	5	81	67	62	48	43

Totals = 369.

Fig. 25.

The foregoing methods for building odd magic squares by a continuous process, involving the regular spacing of consecutive numbers varied by different well defined break moves is believed to be new and original with the writer, but other methods of construction have been known for many years.

One of the most interesting of these older methods involves the use of two or more primary squares, the sums of numbers in similarly located cells of which constitute the correct numbers for transfer into the corresponding cells of the magic square that is to be constructed therefrom.

This method has been ascribed primarily to De la Hire but has been more recently improved by Prof. Scheffler.

It may be simply illustrated by the construction of a few 5×5 squares as examples. Figs. 26 and 27 show two simple primary squares in which the numbers 1 to 5 are so arranged that like numbers occur once and only once in similarly placed cells in the two squares; also that pairs of unlike numbers are not repeated in the same order in any similarly placed cells. Thus, 5 occupies the extreme right hand cell in the lower line of each square, but this com-

bination does not occur in any of the other cells. So also in Fig. 27 3 occupies the extreme right hand cell in the upper line, and in Fig. 26 this cell contains 5. No other cell, however, in Fig. 27 that contains 3 corresponds in position with a cell in 26 that contains 5. Leaving the numbers in Fig. 26 unaltered, the numbers in Fig. 27 must now be changed to their respective key numbers, thus producing the key square shown in Fig. 28. By adding the cell numbers of the primary square Fig. 26 to the corresponding cell numbers

Prime numbers,...1, 2, 3, 4, 5.
Key numbers,.....0, 5, 10, 15, 20.

1	2	3	4	5
1	2	3	4	5
1	2	3	4	5
1	2	3	4	5
1	2	3	4	5

Fig. 26.

1	4	2	5	3
4	2	5	3	1
2	5	3	1	4
5	3	1	4	2
3	1	4	2	5

Fig. 27.

0	15	5	20	10
15	5	20	10	0
5	20	10	0	15
20	10	0	15	5
10	0	15	5	20

Fig. 28.

1	17	8	24	15
16	7	23	14	5
6	22	13	4	20
21	12	3	19	10
11	2	18	9	25

Fig. 29.

of the key square Fig. 28, the magic square shown in Fig. 29 is formed, which is also identical with the one previously given in Fig. 14.

The simple and direct formation of Fig. 14 may be thus compared with the De la Hire method for arriving at the same result.

It is evident that the key square shown in Fig. 28 may be dispensed with by mentally substituting the key numbers for the prime

numbers given in Fig. 27 when performing the addition, and by so doing only two primary squares are required to construct the magic square. The arrangement of the numbers 1 to 5 in the two primary squares is obviously open to an immense number of variations, each of which will result in the formation of a different but perfect magic square. Any of these squares, however, may be more readily constructed by the direct methods previously explained.

A few of these variations are given as examples, the key numbers remaining unchanged. The key square Fig. 32 is formed from the primary square Fig. 31, and if the numbers in Fig. 32 are added to those in the primary square Fig. 30, the magic square given in Fig. 33 will be produced. This square will be found identical with that shown in Fig. 15.

1	1	1	1	1
2	2	2	2	2
3	3	3	3	3
4	4	4	4	4
5	5	5	5	5

Fig. 30.

1	5	4	3	2
3	2	1	5	4
5	4	3	2	1
2	1	5	4	3
4	3	2	1	5

Fig. 31.

0	20	15	10	5
10	5	0	20	15
20	15	10	5	0
5	0	20	15	10
15	10	5	0	20

Fig. 32.

1	21	16	11	6
12	7	2	22	17
23	18	13	8	3
9	4	24	19	14
20	15	10	5	25

Fig. 33.

Fig. 30 cannot be used as a key square, but if two primary squares are constructed in which every horizontal and perpendicular column contains the numbers 1 to 5 placed according to rules previously given, and having a different arrangement of numbers in each primary square, then either of these squares may be made

the key square, and two different magic squares may be constructed therefrom, as shown in the next examples.

The magic square shown in Fig. 37 is made by the addition of numbers in the primary square Fig. 34 to the numbers occupying similar cells in the key square Fig. 36, the latter being derived from the primary square Fig. 35. If the key square shown in Fig. 38 is now constructed from the primary square Fig. 34 and the key numbers therein added to the prime numbers in Fig. 35, the magic square shown in Fig. 39 is obtained. This square has not been given before in this treatise, but it may be directly produced by

3	1	4	2	5
5	3	1	4	2
2	5	3	1	4
4	2	5	3	1
1	4	2	5	3

Fig. 34.

2	1	5	4	3
1	5	4	3	2
5	4	3	2	1
4	3	2	1	5
3	2	1	5	4

Fig. 35.

5	0	20	15	10
0	20	15	10	5
20	15	10	5	0
15	10	5	0	20
10	5	0	20	15

Fig. 36.

8	1	24	17	15
5	23	16	14	7
22	20	13	6	4
19	12	10	3	21
11	9	2	25	18

Fig. 37.

10	0	15	5	20
20	10	0	15	5
5	20	10	0	15
15	5	20	10	0
0	15	5	20	10

Fig. 38.

12	1	20	9	23
21	15	4	18	7
10	24	13	2	16
19	8	22	11	5
3	17	6	25	14

Fig. 39.

an elongated knight's move consisting of two cells to the right and two downwards, using the normal knight's move of two cells to the left and one cell downwards as a break move at every block in the regular spacing.

It will be observed in all the preceding examples that the number 3 invariably occupies the center cell in every primary square, thus bringing 10 in the center of all key squares, and 13 in the center of magic squares, no other number being admissible in the

center cell of a 5×5 magic square. A careful study of these examples should suffice to make the student familiar with the De la Hire system for building odd magic squares, and this knowledge is desirable in order that he may properly appreciate the more direct methods which have been described.

Before concluding this branch of the subject, mention may be made of another method for constructing odd magic squares which is said to have been originated by Bachet de Mezeriac. The application of this method to a 5×5 square will suffice for an example.

The numbers 1 to 25 are written consecutively in diagonal columns, as shown in Fig. 40, and those numbers which come out-

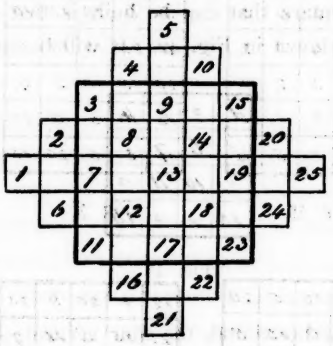


Fig. 40.

3	16	9	22	15
20	8	21	14	2
7	25	13	1	19
24	12	5	18	6
11	4	17	10	23

Fig. 41.

side the center square are transferred to the empty cells on the opposite sides of the latter without changing their order. The result will be the magic square of 5×5 shown in Fig. 41. It will be seen that the arrangement of numbers in this magic square is similar to that in the 7×7 square shown in Fig. 4, which was built by writing the numbers 1 to 49 consecutively according to rule. The 5×5 square shown in Fig. 41 may also be written out directly by the same rule without any preliminary or additional work.

EVEN MAGIC SQUARES.

In perfect squares of this class it is necessary that the sum of each column shall be the same amount, and also that the sum of

any two numbers that are geometrically equidistant from the center of the square shall equal the sum of the first and last numbers of the series.

The numbers in the two corner diagonal columns in even magic squares may be determined by writing the numbers of the series in arithmetical order in horizontal columns, beginning with the first number in the left hand cell of the upper line and writing line after line as in a book, ending with the last number in the right hand cell of the lower line. The numbers then found in the two diagonal columns will be in magic square order, but the position of the other numbers must generally be changed.

The smallest even magic square that can be built is that of 4×4, and one of its forms is shown in Fig. 42. It will be seen

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

Fig. 42.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Fig. 43.

that the sum of each of the four horizontal, the four vertical, and the two corner diagonal columns in this square is 34, making in all ten columns having that total; also that the sum of any two geometrically opposite numbers is 17, which is the sum of the first and last numbers of the series. It is therefore a perfect square of 4×4.

The first step in the construction of this square is shown in Fig. 43, in which only the two corner diagonal columns, which are written in heavy figures, have the correct summation. The numbers in these two columns must therefore be left as they are, but the location of all the other numbers, which are written in light figures, must be changed. A simple method for effecting this change consists in substituting for each number the complement between it and 17. Thus, the complement between 2 and 17 is 15, so 15 must be written in the place of 2, and so on throughout. All of the light figure

numbers being thus changed, the result will be the perfect magic square shown in Fig 42.

The same relative arrangement of figures may be attained by leaving the light figure numbers in their original positions as shown in Fig. 43, and changing the heavy figure numbers in the two corner diagonal columns to their respective complements with 17. It will be seen that this is only a reversal of the order of the figures

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

Fig. 44.

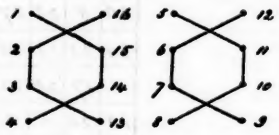


Fig. 45..

in the two corner diagonal columns, and the resulting magic square which is shown in Fig. 44 is simply an inversion of Fig. 42.

Fig. 45 is a geometrical diagram of the numbers in Fig. 42, and it indicates a regular law in their arrangement, which also holds good in many larger even squares, as will be seen later on.

There are many other arrangements of sixteen numbers which will fulfil the required conditions but the examples given will suffice to illustrate the principles of this square.

1	35	34	3	32	6
30	8	28	27	11	7
24	23	15	16	14	19
13	17	21	22	20	18
12	26	9	10	29	25
31	2	4	33	5	36

Fig. 46.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

Fig. 47.

The next even magic square is that of 6×6, and one of its many variations is shown in Fig. 46. An analysis of this square with the aid of geometrical diagrams will point the way not only

to its own reconstruction but also to an easy method for building 6×6 squares in general.

Fig. 47 shows a 6×6 square in which all the numbers from 1 to 36 are written in arithmetical sequence, and the twelve numbers in the two corner diagonal columns will be found in magic square order, all other numbers requiring rearrangement. Leaving there-

1	35	34	33	32	6
30	8	28	27	11	25
24	23	15	16	20	19
18	17	21	22	14	13
12	26	10	9	29	7
31	5	4	3	2	36

Fig. 48.

fore the numbers in the diagonal columns unchanged, the next step will be to write in the places of the other numbers their complements with 37, making the square shown in Fig. 48. In this square twenty-four numbers (written in heavy figures) out of the total of

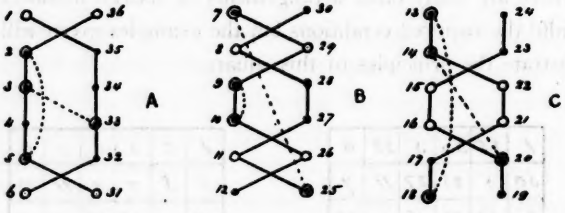


Fig. 49.

thirty-six numbers, will be found in magic square order, twelve numbers (written in light figures) being still incorrectly located. Finally, the respective positions of these twelve numbers being reversed in pairs, the magic square given in Fig. 46 will be produced.

Fig. 50 shows the geometrical diagrams of this square, A being a diagram of the first and sixth lines, B of the second and fifth lines, and C of the third and fourth lines. The striking ir-

regularity of these diagrams points to the imperfection of the square which they represent, in which, although the sum of each of the two corner diagonal, the six horizontal, and the six perpendicular columns is 111, yet only in the two diagonal columns does the sum of any two numbers which occupy geometrically opposite cells, amount to 37, or the sum of the first and last numbers of the series. Owing to their pronounced irregularities, these diagrams convey but little meaning, and in order to analyze their value for further constructive work it will be necessary to go a step backwards and make diagrams of the intermediate square Fig. 48. These diagrams are shown in Fig. 49, and the twelve numbers therein which must be transposed (as already referred to) are marked by small circles around dots, each pair of numbers to be transposed in position

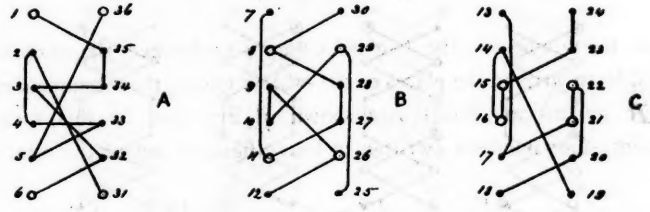


Fig. 50.

being connected by a dotted line. The numbers in the two corner diagonal columns which were permanently located from the beginning are marked with small circles.

We have here correct geometrical figures with definite and well defined irregularities. The series of geometrical figures shown in A, B, and C remain unchanged in shape for all variations of 6×6 squares, but by modifying the irregularities we may readily obtain the data for building a large number of different 6×6 squares, all showing, however, the same general characteristics as Fig. 46.

A series of these diagrams, with some modifications of their irregularities, is given in Fig. 51, and in order to build a variety of 6×6 magic squares therefrom it is only necessary to select three diagrams in the order A, B, and C, *which have each a different form*

of irregularity, and after numbering them in arithmetical sequence from 1 to 36, as shown in Fig. 49, copy the numbers in *diagrammatic order* into the cells of a 6×6 square.

It must be remembered that the cells in the corner diagonal columns of even magic squares may be correctly filled by writing

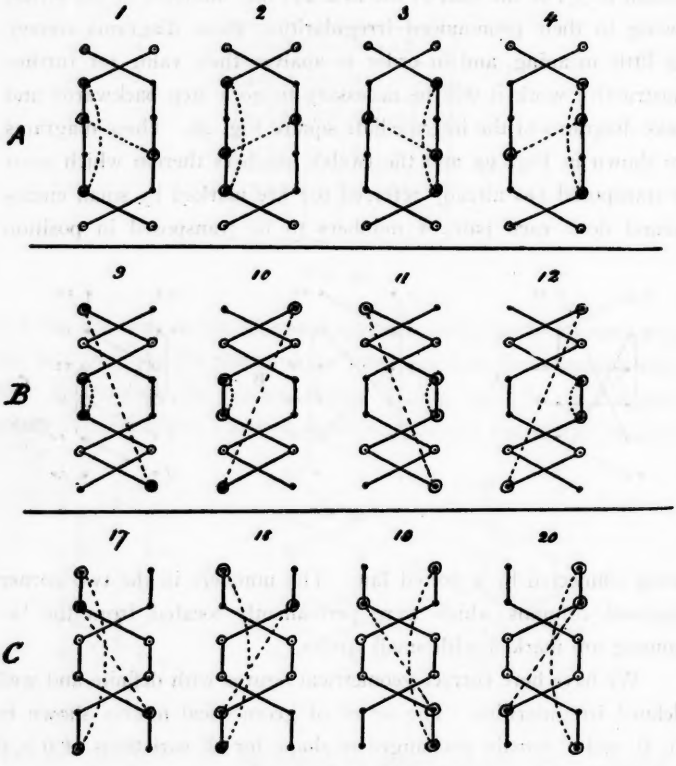


Fig. 51 (First Part).

the numbers in arithmetical order according to the rule previously given, so in beginning any new even square it will be found helpful to first write the numbers in these columns, and they will then serve as guides in the further development of the square.

Taking for example the 6×6 magic square shown in Fig. 46,

it will be seen from Fig. 49 that it is constructed from the diagrams marked 1—9 and 17 in Fig. 51. Comparing the first line of Fig. 46 with diagram A, Fig. 49, the sequence of numbers is 1,—35,—34 in unbroken order; then the diagram shows that 33 and 3 must be transposed, so 3 is written next (instead of 33) then 32 and 6 in

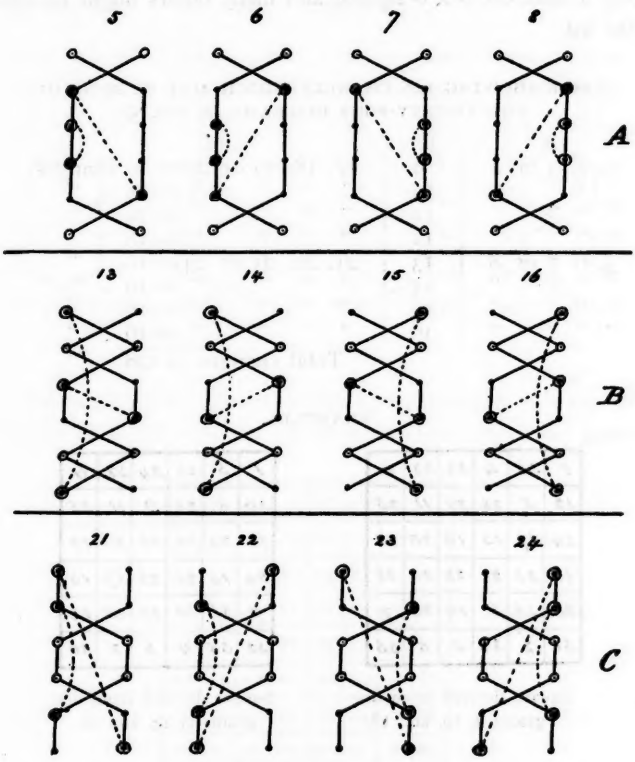


Fig. 51 (Second Part).

unbroken order. In the last line of this square (still using diagram A) 31 comes first, then, seeing that 5 and 2 must be transposed, 2 is written instead of 5; then 4; then as 3 and 33 must be transposed, 33 is written instead of 3, 5 instead of 2, and the line is finished with 36. Diagram B gives the development of the second

and fifth lines of the square in the same manner, and diagram C the development of the third and fourth lines, thus completing the square.

The annexed table shows 128 changes which may be rung on the twenty-four diagrams shown in Figure 51, each combination giving a different 6×6 square, and many others might be added to the list.

TABLE SHOWING 128 CHANGES WHICH MAY BE RUNG ON THE TWENTY-FOUR DIAGRAMS IN FIG. 51.

A	B	C
1, 2, 3 or 4	9	17, 18, 19 or 20 = 16 changes
" " " "	10	" " " " = 16 "
" " " "	11	" " " " = 16 "
" " " "	12	" " " " = 16 "
5, 6, 7 or 8	13	21, 22, 23 or 24 = 16 "
" " " "	14	" " " " = 16 "
" " " "	15	" " " " = 16 "
" " " "	16	" " " " = 16 "
		Total changes = 128 "

EXAMPLES.

1	35	4	33	32	6
12	8	28	27	11	25
24	17	15	16	20	10
13	23	21	22	14	18
30	26	9	10	29	7
31	2	34	3	5	36

Square derived from diagrams 2, 10, and 18.

1	5	33	34	32	6
30	8	28	9	11	25
18	23	15	16	20	19
24	14	21	22	17	13
7	26	10	27	29	12
31	35	4	3	2	36

Square derived from diagrams 8, 13, and 22.

The next size of even magic square is that of 8×8, and instead of presenting one of these squares ready made and analyzing it, we will now use the information which has been offered by previous examples in the construction of a new square of this size.

Referring to Fig. 45, the regular geometrical diagrams of the 4×4 square naturally suggest that an expansion of the same may be utilized to construct an 8×8 square. This expanded diagram

is accordingly shown in Fig. 52, and in Fig. 53 we have the magic square that is produced by copying the numbers in diagrammatic order.

As might be anticipated, this square is perfect in all its characteristics, and the ease with which it has been constructed points to the simplicity of the method employed.

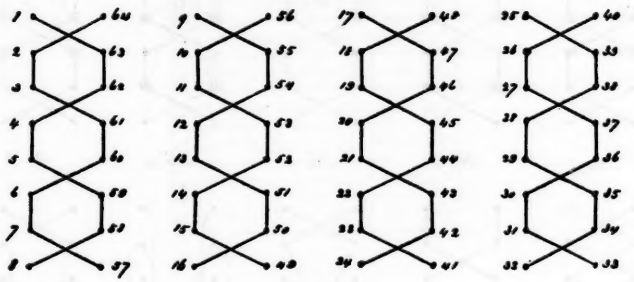


Fig. 52.

The magic square shown in Fig. 53 is, however, only one of a multitude of 8×8 squares, all of which have the same general

1	63	62	4	5	59	58	8
56	10	11	53	52	14	15	49
48	18	19	45	44	22	23	41
25	39	38	28	29	35	34	32
33	31	30	36	37	27	26	40
24	42	43	21	20	46	47	17
16	50	51	13	12	54	55	9
57	7	6	60	61	3	2	64

Totals = 260.

Fig. 53.

characteristics and may be constructed with equal facility from various regular diagrams that can be readily derived from transpositions of Fig. 52. Five of these variations are illustrated in Fig. 54, which also show the transpositions by which they are formed from the original diagrams. To construct a perfect magic square

from either of these variations it is only necessary to make four copies of the one selected, annex the numbers 1 to 64 in arithmetical

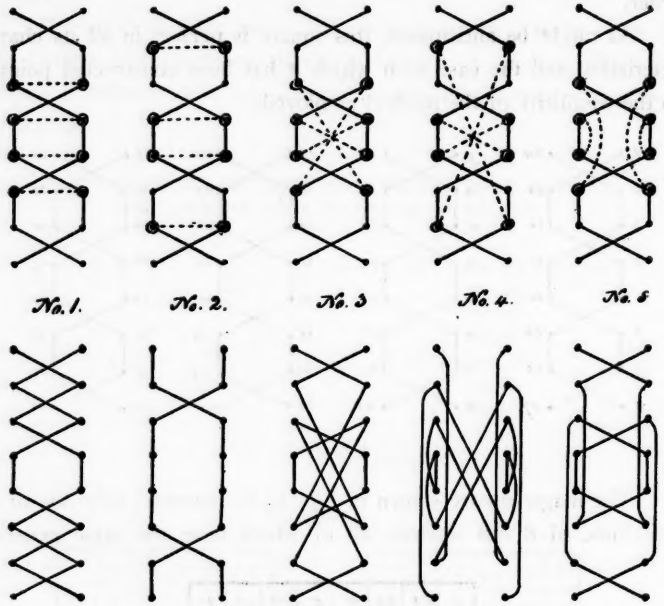


Fig. 54.

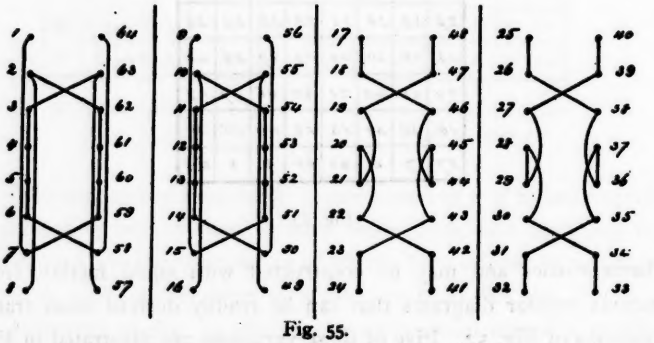


Fig. 55.

order as before explained, and then copy the numbers in diagrammatic sequence into the cells of an 8×8 square.

It will be noted in the construction of the 4×4 and 8×8 squares that only one form of diagram has been hitherto used for each square, whereas three different forms were required for the 6×6 square. It is possible, however, to use either two, three, or

1	7	59	60	61	62	2	8
16	10	54	53	52	51	15	9
48	47	19	21	20	22	42	41
33	34	30	28	29	27	39	40
25	26	38	36	37	35	31	32
24	23	43	45	44	46	18	17
56	50	14	13	12	11	55	49
57	63	3	4	5	6	58	64

Totals = 260.

Fig. 56.

four different diagrams in the construction of an 8×8 square, as shown in the annexed examples. Fig. 55 illustrates two different forms from which the magic square Fig. 56 is constructed. Fig. 57

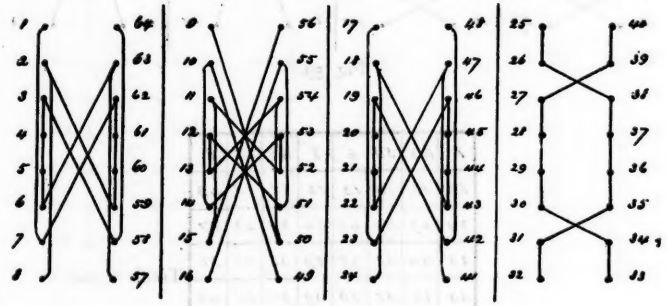


Fig. 57

shows three different forms which are used in connection with the square in Fig. 58, and in a similar manner Figs. 59 and 60 show four different diagrams and the square derived therefrom. The foregoing examples are sufficient to illustrate the immense number

1	7	62	61	60	59	2	8
49	10	14	53	52	11	15	56
48	42	19	20	21	22	47	41
40	39	27	28	29	30	34	33
32	31	35	36	37	38	26	25
24	18	43	44	45	46	23	17
9	50	54	13	12	51	55	16
57	63	6	5	4	3	58	64

Totals = 260.

Fig. 58.

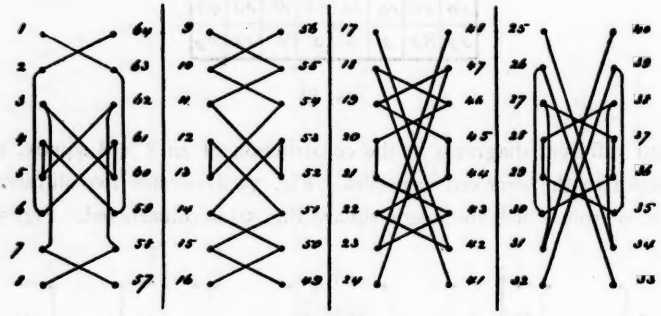


Fig. 59.

1	63	59	4	5	62	58	8
56	10	54	13	12	51	15	49
24	47	19	45	44	22	42	17
25	34	38	28	29	35	39	32
33	26	30	36	37	27	31	40
48	23	43	21	20	46	18	41
16	50	14	53	52	11	55	9
57	7	3	60	61	6	2	64

Totals = 260.

Fig. 60.

of different 8×8 magic squares that may be constructed by the aid of various diagrams.

We now come to the magic square of 10×10, and employing the comparative method of the last examples, it will be easy to expand the three diagrams of the 6×6 square (Fig. 49) into five

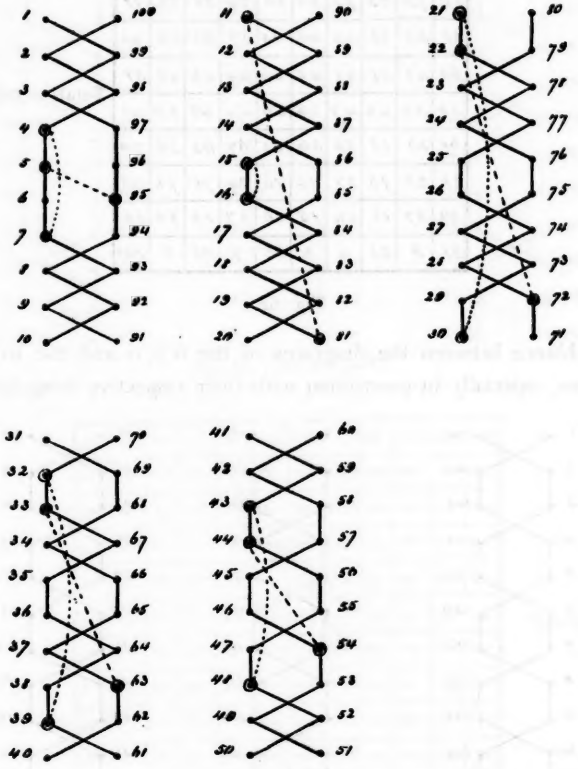


Fig. 61.

diagrams that are required for the construction of a series of 10×10 squares. These five diagrams are shown in Fig. 61, and in Fig. 62 we have the magic square which is made by copying the numbers from 1 to 100 in diagrammatic order into the cells of a 10×10 square.

It will be unnecessary to proceed further with the construction of other 10 × 10 squares, for the student will recognize the striking

1	92	3	97	96	5	94	8	92	10
90	12	88	14	86	85	17	83	19	11
80	79	23	77	25	26	74	28	22	71
31	69	68	34	66	65	37	33	62	40
60	42	58	57	45	46	44	53	49	51
50	52	43	47	55	56	54	48	59	41
61	32	38	64	36	35	67	63	39	70
21	29	73	27	75	76	24	78	72	30
20	82	18	84	15	16	87	13	89	81
91	9	93	4	6	95	7	98	2	100

Totals = 505.

Fig. 62.

resemblance between the diagrams of the 6 × 6 and the 10 × 10 squares, especially in connection with their respective irregularities,

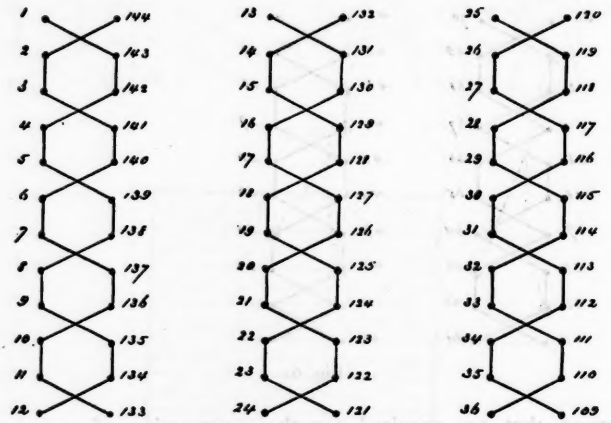


Fig. 63 (First part).

which point to the apparent impossibility of building perfect 10 × 10 squares.

It will also be seen that the same methods which were used for

varying the 6×6 diagrams, are equally applicable to the 10×10 diagrams, so that an almost infinite variety of changes may be rung on them, from which a corresponding number of 10×10 squares

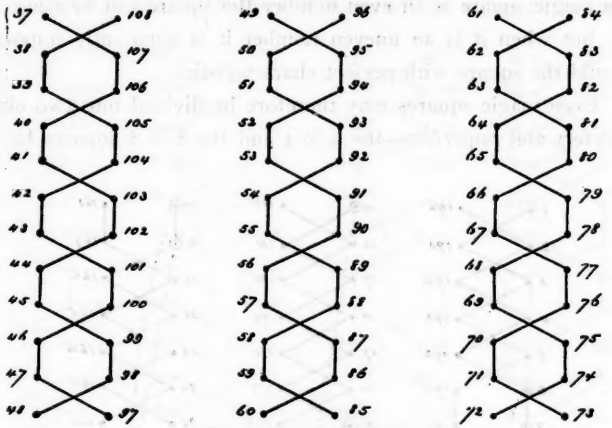


Fig. 63 (Second part).

1	143	142	4	5	139	138	8	9	135	134	12
132	14	15	129	128	18	19	125	124	22	23	121
120	26	27	117	116	30	31	113	112	34	35	109
37	107	106	40	41	103	102	44	45	98	98	48
49	95	94	52	53	91	90	56	57	87	86	60
84	62	63	81	80	66	67	77	76	70	71	73
72	74	75	69	68	78	79	65	64	82	83	61
85	59	58	88	89	55	54	92	93	51	50	96
97	47	46	100	101	43	42	104	105	39	38	108
36	110	111	33	32	114	115	29	28	118	119	25
24	122	123	21	20	126	127	17	16	130	131	13
133	11	10	136	137	7	6	140	141	3	2	149

Totals = 870

Fig. 64.

may be derived, each of which will be different but will resemble the series of 6×6 squares in their curious and characteristic imperfections.

We have thus far studied the construction of all even magic squares up to and including that of 10 × 10, and it is worthy of remark that when one half the number of cells in one side of an even magic square is an even number the square can be made perfect, but when it is an uneven number it is apparently impossible to build the square with perfect characteristics.

Even magic squares may therefore be divided into two classes —perfect and imperfect—the 4 × 4 and the 8 × 8 squares belong-

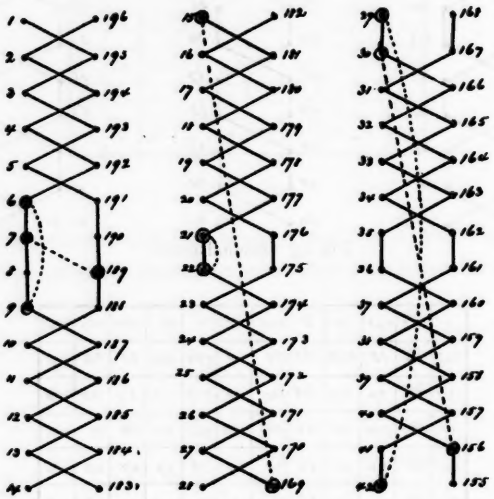


Fig. 65 (First Part).

ing to the first class, and the 6 × 6 and 10 × 10 belonging to the second class.

Fig. 63 shows a series of diagrams from which the 12 × 12 square in Fig. 64 is derived. The geometrical design of these diagrams is the same as that shown in Fig. 52 for the 8 × 8 square, and it is manifest that all the variations that were made in the 8 × 8 diagrams are also possible in the 12 × 12 diagrams, besides an immense number of additional changes which are allowed by the increased size of the square.

In Fig. 65 we have a series of diagrams illustrating the development of the 14×14 magic square shown in Fig. 66. These diagrams being plainly derived from the diagrams of the 6×6 and 10×10 squares, no explanation of them will be required, and it is

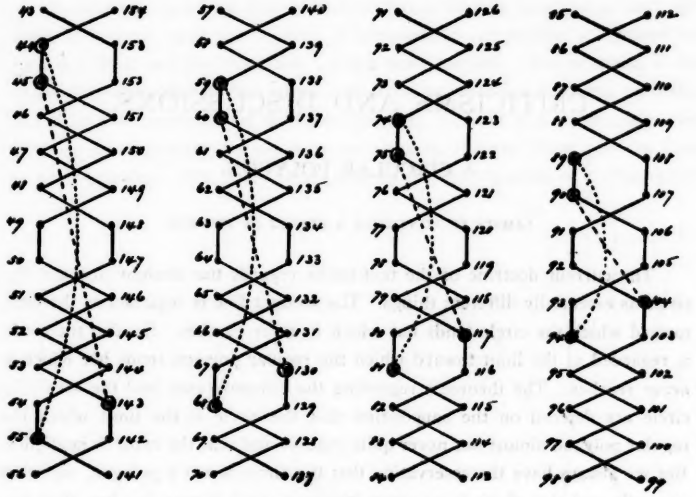


Fig. 65 (Second Part).

evident that the diagrammatic method may be readily applied to the construction of all sizes of even magic squares.

[TO BE CONCLUDED.]

W. S. ANDREWS.

NEW YORK.

CRITICISMS AND DISCUSSIONS.

A CIRCULAR POLYGON.

LIMITS IN CONTINUA A RESULT OF PROCESS.

The current doctrine of the text-books regards the straight line and the circle as essentially different things. The straight line is regarded as the limit toward which the circle tends but which it never reaches. So also the circle is regarded as the limit toward which the regular polygon tends but which it *never* reaches. The theorems regarding the circumference and the area of a circle are derived on the supposition that the circle is the limit which the regular polygon almost but never quite reaches and that the error is negligible. But we always have the reservation that the circle is *not* a polygon, say what you will, and that there *is* an error, however small it may be; less than any assignable quantity, but yet an error after all. The difference between the circle and polygon is so small that for all practical purposes we may consider them as one, but of course they are not one, and never can be, etc., etc.

And through all the array of verbiage we feel that there is a fallacy somewhere; it is and it isn't, all in the same breath; the error is inexpressible and yet the two forms do not coincide. We can push the polygon almost to the circle; what is that invisible barrier which keeps it back?

There is no barrier except our own narrow definitions and methods. The straight line is a circle of special form, not the limit of a circle; the circle is a polygon, not the limit of a polygon. There is no error. The circle straightens out into a straight line and sweeps over it into a circle on the other side. The inscribed polygon merges into the circle and sweeps over it into a polygon again on the other side.

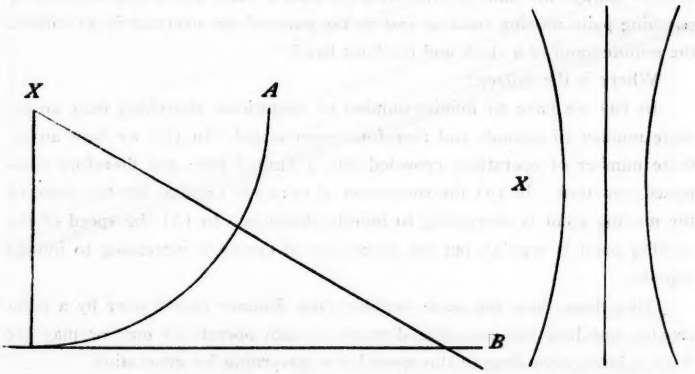
Let us see what is meant by a limit, and why forms have limits.

A limit is that constant value (or form) which a variable value (or form) approaches indefinitely near but never quite reaches. The test of a limit is, where r is the limit of x, $r-x=0$, and $r-x<i$, where i is any infinitely small value.

The subject of limits as taught in the elementary text-books is very crude and fogged with lack of perspective.

In the first place no distinction is made between the limit in the case of geometric forms (continua) and that of numbers (discreta). The two cases are quite different and the distinction must be recognized.

It seems to be a rule that geometric forms have or have not a limit, dependent entirely upon the method of generation; one method of generation having a limit and the other not, for the same variable. For example, if the angle X is generated by the movement of the intersection A, it has no limit; but if by the movement of the intersection B, its limit is a right angle. So likewise if we generate the arc s by the swelling of a cartwheel rim, the limit is a straight line. But if we generate it by the tracing-point of a Peaucellier



linkage, it has no limit; it straightens out into a straight line and then curves the other way. In both these cases, it is the same variable, a line of constant curvature. The elementary text-books blindfold their readers with a, not necessarily faulty, but narrow definition. A circle is a line which always changes its direction and a straight line is one which does not change etc. And then triumphantly ask how one can be the other.

Throw away the blinders and get a broader view by taking a broader definition, viz., a line of constant curvature, and the contradiction ends.

The old contradiction between a tangent and a secant has begun its evanescence by considering the tangent as a secant cutting in two coincident points, one double point.

But when it comes to swelling an inscribed polygon into a circle, then, they say, the law laid down above fails, since there is no instrument to do the swelling, and however far you continue the process, there are points of circumference yet unoccupied by vertices of the polygon. The same objection

would have been made in the case of the circle and straight line, previous to 1864 when Peaucellier invented his linkage; showing that the question of limit does not depend upon the inventiveness of man; but only our appreciation. Such an instrument could have been imagined *in nubibus*, and the same argument used as here, and the argument would have been just as sound. The only difference would have been its effect upon the hearers.

Let us look at some examples of limits.

(*a*) A point moving half the distance remaining between itself and the goal each second; when will it reach the goal? Never, because between it and the goal will ever remain the half of some distance. (*b*) A point moving away half the distance between it and a pursuing point each instant of time; when will the pursuing point catch the other? Never, because the pursuing point is always the half of some distance ahead. But this is nonsense, for a pursuing point moving twice as fast as the pursued can overtake it, as witness the minute hand of a clock and the hour hand.

Where is the fallacy?

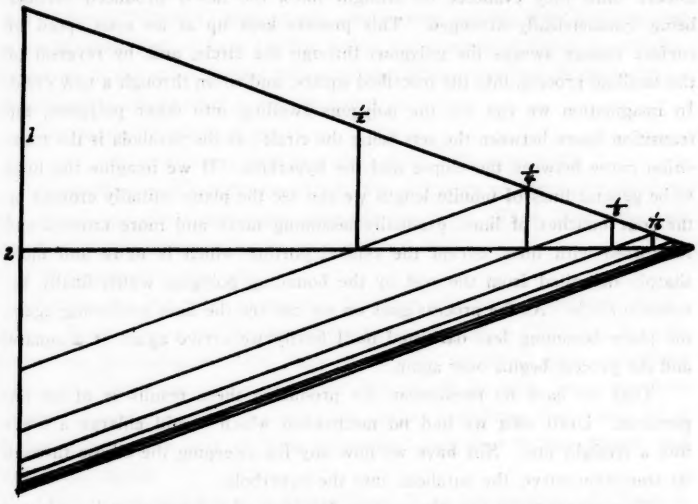
In (*a*) we have an infinite number of operations, stretching over an infinite number of seconds and therefore never ended. In (*b*) we have an infinite number of operations crowded into a limited time and therefore completed sometime. In (*a*) the succession of events is regular, but the speed of the moving point is decreasing to infinite slowness. In (*b*) the speed of the moving point is regular, but the succession of events is increasing to infinite rapidity.

This shows how the same variable (the distance passed over by a point moving one half the preceding distance at each operation) may or may not have a limit, according to the special law governing its generation.

The introduction of a *timed succession* of events (finite intervals) produces a decreasing speed and a limit. A timed (finite intervals or constant) speed produces an increasing rapidity of succession of events and no limit. A horse straining at his halter finds the distance between him and the door diminished one half each second. Can he ever get out? Never! A horse straining at his halter finds the distance between him and the door diminished one half at each instant of strain. Can he get out? Certainly, a steady strain will carry him through the door. In the first case there is a timed succession of events. In the second case there is a continuous and steady strain, a timed rate of progress, finite speed.

In the case of the summation of a series of discrete terms the use of discrete terms seems to be the equivalent of a timed succession of events, and the series has a limit, if convergent. An illustration of the difference between the summation of discreta and continua is given in the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$. If we consider these terms as ordinates erected at finite intervals, the summation has a limit, 2. But if we crowd the ordinates into a triangle

as shown, the sum is easily seen, by reason of the similar triangles to be exactly 2, and there is no unattainable limit, no residual error. We actually reach the sum, 2. In each case we have dealt with *exactly the same ordinates;* and in one case *arranged* so as to have a limit to the sum, in the other case



no limit. Which result we shall get is entirely a question of arrangement. In this instance, and for other geometric series, the sum of an infinite converging series is a real, tangible quantity, and not an elusive limit, just out of reach. A limit is the limit of a process and not necessarily any intrinsic property of the variable itself.

If we imagine the inscribed polygon swelled toward the circle by doubling the number of sides, etc., the circle seems to be the limit of the operation, for the process of doubling introduces the timed succession of events which results in a limit. But imagine a process which forced each centre of a chord (inscribed square) into a symmetrical position (i. e. on to the circumference of the circle, through the undisturbed points) and this process kept up at an *even speed of surface change,* like the constant strain of the horse at his halter, or the even movement of the minute hand. The succession of events increases to infinite rapidity, and the inscribed polygons sweep through the circle into the circumscribed polygons. The newly produced vertices are arranged on the circumference of the initial circumscribing circle until the circle itself is reached, and then they arrange themselves on circles (of increasing sizes) the alternate vertices being forced out until the undisturbed ones evanesce on straight lines and the polygons become of lessening number

of sides until the circumscribing square is reached, and the process repeats itself into a new circle around this new square, and so on. If instead of saying "alternate vertices etc.," suppose we say, reversal of the process which produces the circle from the circumscribed square by forcing the vertices inward until they evanesce on straight lines, the newly produced vertices being symmetrically arranged. This process kept up at an even speed of surface change sweeps the polygons through the circle, and, by reversal of the swelling process, into the inscribed square, and so on through a new cycle. In imagination we can see the polygons swelling into other polygons, the transition figure between the sets being the circle; as the parabola is the transition curve between the ellipse and the hyperbola. If we imagine the lines to be general lines of infinite length we can see the plane, initially crossed by the four bunches of lines, gradually becoming more and more crossed and re-crossed with lines, except the central portion which is more and more sharply delimited from the rest by the bounding polygon, which finally becomes a circle. As the process goes on we can see the lines coalescing again, the plane becoming less darkened until finally we arrive again at a square, and the process begins over again.

That we have no mechanism for producing these results is of no importance. Until 1864 we had no mechanism which would enlarge a circle into a straight line. Nor have we now any for sweeping the ellipse through its transition curve, the parabola, into the hyperbola.

To conform to all this, the narrow definition of polygon must be enlarged into a configuration of lines; a regular polygon as a symmetrical configuration, one phase of which is the circle, a regular configuration of an infinite number of lines.

A pregnant illustration of this idea is the historic problem of squaring the circle. In numbers this is impossible because π, among other reasons, is the limit of an infinite series of discrete terms. In geometry, with a ruler and compass, the length of the circumference is also the limit of an infinite series of operations and is therefore unattainable. But change the process by using the integraph, and what was before a limit and just out of reach becomes attainable, and we get a straight line equal in length to the circumference.

ARTHUR LATHAM BAKER.

MANUAL TRAINING HIGH SCHOOL, BROOKLYN, N. Y.

A MOSLEM ACCOUNT AS TO THE ORIGIN OF CHRISTIAN SECTS IN SYRIA.

To the Editor of The Monist.

The account of Christianity in the work of Shahrastáni discussed in *The Monist* of January last, more especially the concluding paragraph (p. 122 inf.),

alluding to three chief sects of Christians in Syria, reminds me of a certain extract from Mirkhond (Mírkhavand), which somewhat puzzles me, and which no one to whom I have applied here has been inclined or able to explain to my satisfaction.

I offer a translation from the Persian original (*Raudat-al-safá*, or "Garden of Purity," Book I) without any commentary, leaving it to you what use you will make of it. I have tried to follow the original as closely as possible, especially in the characteristic terms, among which "the God of the earth"—*khudát-zamin*—has rather a strange sound....

The passage reads as follows:

"OF YÚNUS THE JEW LEADING ASTRAY THE CHRISTIANS.

"According ot Khamdín-ibn-'Abbás the followers of Jesus remained on the sacred path for eighty years after his ascension. Then Yúnus the Jew led them away into the valley of error and unbelief.

"It happened in this wise:

"Yúnus the Jew came among the Christians wearing a hermit's or monk's gown. Four months did he stay at the house of a pious Christian, showing to no one his unblessed face. His devout life produced great confidence among the Christians, and when, after that period, he said, 'Send me three of your learned men, in whom you place perfect confidence, that I may lay a divine secret before each of them separately,' they sent to him Nestor and Yakúb and Malek.

"And in private he said to one of them, 'I am a messenger of Christ to the people, to lighten the burden of their hearts on account of (*sic*!) his word.' And further he said, 'Do you know that Jesus restored life to the dead, and did such and such things?' The learned man replied: 'Aye.' Then Yúnus asked him, 'Could any one but God do such things?' He said, 'No.' Yúnus said, 'Know then for certain, that Jesus is the Providence of that world, who came down from heaven, and having borne his earthly lot to the end, went back to heaven.'

"To the second learned man he said, privately, 'Are you aware that the Lord Jesus performed such acts as no one but the Lord Creator could have done?' The other assented. Yúnus said, 'Do you know that the Lord Jesus is sinless?' He said, 'Aye.' Yúnus said, 'Then you ought to believe that Jesus is the son of God, who sent him to the earth and took him up again into heaven.'

"And with the third sage, again in private, he spoke similar words, saying, 'He is the God of the earth, who, when men intended to kill him, was concealed, and who will shortly return among the people. I have been sent to bring these tidings.'

"Having communicated to them such false doctrine, he went into the oratory, where he locked himself in. But that very night he went the way

to hell. And on the morrow, when the Christians inquired from the three learned men what Yúnus spoke to them, each of them had a different message from the other two. So the people said, 'We had better hear Yúnus himself.' And they went and burst open the door of the oratory and found that Yúnus had been killed. And the Christians were divided into three sects, each one adopting one of the three before-mentioned creeds."

H. WERNEKKE.

WEIMAR, GERMANY, February 27, 1905.

EDITORIAL COMMENT.

It appears that the communication of Professor Wernekke, ultimately based upon the report of Khamdín-ibn-'Abbás, is the ancient Moslem view as to the origin of Christian sects. According to Mohammed, Jesus was born of a virgin, and is the greatest prophet of past ages, but the Christians are accused of misinterpreting the doctrine of their leader. Among other things it is claimed that Jesus when speaking of the Comforter who would come and initiate Christians into the truth (John xv. 26) referred to Mohammed; for they claim that the Greek word παρακλητος (i. e., Comforter) is but a corruption of παρακλυτος (i. e., the Renowned One) which is the meaning of the name Mohammed. The present account is characteristic of the Moslem view. It assumes that the primitive Christians were Christ's true followers, but that their faith was perverted by a scheming villain, and even in the Orient the odium of heresy is laid at the door of the Jews. P. C.

BOOK REVIEWS.

THE COLOR LINE. A Brief in Behalf of the Unborn. By *William Benjamin Smith.* Pp. xv, 261. New York: McClure, Phillips & Co.

It will presently appear that this work is notably interesting on several accounts apart from the gravity of its subject. That the race problem, particularly in the form in which it confronts the American people, is indeed a momentous subject, none perhaps will deny. But who is the author? What are his credentials? Is there good reason to suppose him qualified to undertake so formidable a task? Is he capable of conceiving it adequately in both its abstract and its concrete bearings, in its present and its future significance, as at once a most intricate problem of pure science and a frightfully complicated practical question for the decision of statesmen? Is he one from whom may be expected a really serious, enlightened plea, free alike from ignorance, from provincial prejudice, and from blinding sentiment? Such are the questions that busy men, before giving serious attention to any discussion of a great subject, are naturally wont to ask, and in these crowded days of abounding authorship they are right.

Doubtless many will be interested in the information that the *Color Line* was written by the doubting "'David" of the *Reign of Law,* though in truth it must be said that the "original" of David was much transformed and but little transfigured or glorified by Mr. Allen's fantasy. The book in hand is the first of its kind by a mathematician; and all the qualities of the mathematical mind, excepting that of proverbial dryness, are evident throughout, in its grasp and penetration, in the clearness and steadiness of its vision, in the sharp precision with which its problems are stated, and in the boldness, energy, and relentless logical rigor with which they are handled. Professor Smith, who is head of the mathematical department of Tulane University, is among the first Americans who gained the doctorate at a German University, where he won prizes for brilliant achievement in the austerest of the sciences, physics and mathematics. As an inspiring teacher of "the universal art apodictic" and as a writer on geometry both pure and analytic, and on infinitesimal analysis, his reputation is national. The present book, however.

is not his first achievement outside the domain of mathematics. Neither is it his first essay beyond the limits of purely academic authorship. Far from it. His *Life of James Sidney Rollins, "Pater Universitatis Missouriensis,"* written by request and printed for private distribution, his series of papers in defence of the gold standard—a defence which in view of his Southern residence and professional connection seems scarcely less than heroic, demonstrating both patriotism and spiritual independence — these taken together with his numerous contributions to the discussion of the tariff question, will serve to indicate something of the range of his interest and activity, without pausing to name the chief products of his pen, a series of critical memoirs in the field of New Testament literature that have secured him and American Biblical scholarship the profound respect of critical circles in England, Holland, and Germany.

"They knew not," said Leibnitz of his friends who feared he would specialize too narrowly, "that my mind could not be satisfied with one kind of things." It may indeed be that Leibnitz was, as DeQuincy calls him, "the last of the universals." It is consoling to know, however, that any such fine *mot* is at best but a partial truth, for, if inexpertness is the curse of the world, the defect of expertness is depth at the expense of breadth, and many a larger question has to await the single intelligence that shall be at once deep and *comprehensive*. In this view it is matter for just pride and congratulation that our most difficult social problem has received painstaking study and candid treatment by an American scholar of whom it has been said that in his best moments of well-being he appears to hold the entire body of the intellectual achievements of the race in something very like true perspective. If this seem to be fulsome praise, we admit the appearance but deny the reality. It accords at any rate with our sober judgment deliberately formed in the course of years. And we gladly record the judgment here as valid for us in despite of the fact that the conclusions of the "Brief," while they commend themselves to our understanding, are far from compatible with our liveliest sentiments and severely condemn our former standards of conduct. For we admit, or avow, or confess, as you will, that in the matter of social equality our sympathies have always hitherto favored the standard of *individual* as distinguished from *racial* excellence. We vividly recall, in view of Professor Smith's terrific arraignment, the fearful risk we assumed of social ostracism when as a resident in a Southern community we repeatedly invited to our home a very black negro in order to enjoy the exceeding wit and intelligence and candor of his conversation.

What of it? Well, in so doing, we were entirely unconscious that we were doing wrong; quite the contrary in fact; presumably we were ignorant, we had not reflected. Were we innocent? Far from it, if the doctrines of this book be sound. By these, which in matters ethical transcend the common

categories and align themselves with the solemn secularities of the cosmic processes, our conduct was very wicked; not that any paltry *specimen* of it was especially reprehensible, it was bad in *kind*, for its kind is such as needs but to be generalized, adopted in common practice, to work the irreversible doom of the highest race; we sinned against an *idea*, more sacred even than that of family, our offence was one against "the most sacred thing on earth," the idea and norm of the Caucasian race. For, so the "Brief" contends, it is not merely the privilege, it is the highest duty of this race in the South, in the North, everywhere, "to keep open at all times, at all hazards, and at all sacrifices, an impassable social chasm between the Black and the White, no matter what the virtues or abilities or accomplishments" of any individual Black may chance to be. Why? Because of the *sole* alternative. And what is that? The answer is: miscegenation, mongrelization, race amalgamation, *pammixia*, the deadly enemy of race improvement and chief among the agencies of inheritable organic degeneration.

Such in brief is the cardinal thesis of this brilliant book. Of its correctness the author obviously entertains not an appreciable doubt, but he does not mistake his own conviction for that of others; and accordingly, against every conceivable objection and counter hypothesis, he defends his doctrine with a splendid army of fact and argument assembled from every department of the commonwealth of knowledge. History and biology, literature and psychology, mathesis and art, philosophy, anthropology, sociology, and statistics, all are made to contribute to the defence of his position. Is the defence everywhere successful? As to that readers will doubtless differ, but it is entirely safe to affirm that the assailing forces are sadly in need of repose and of new recruits and better discipline.

That race amalgamation *is* the inevitable corollary of social equality is the testimony of observation and common sense alike, and is proved by the uncontradicted witness of history. That amalgamation of two organically unequal races yields a product organically inferior to the superior race is the doctrine at once of history and biology. That the Negro is, not merely a "backward race," but really, organically, mentally, hopelessly inferior to the White, is a proposition supported, not alone by the best ascertained of facts biological, ethnological, and anthropological, but by the total immensity, "the infinite variety, of consentient testimony of all historic time and place." These propositions once granted, the author's main thesis follows beyond a doubt; nay, it fairly leaps into the field of vision, not merely however as a truth to be beheld but as a mistress of conduct, a commander of conscience. Their establishment is, therefore, the principal performance, the chief contribution of the book.

Incidentally, however, and by way of supplement, other weighty words are spoken. We do not remember to have seen elsewhere a weightier or

juster deliverance concerning the inherent limitations of the power of education than that contained in Chapter V. Let the following serve as example:

"It is a colossal error to suppose that race improvement, in the strictest sense of the term, can be wrought by education. The reason is simple and easily understood: Race-improvement is organic; education is contra-organic. Any change or amelioration that affects the race, the stock, the blood, must be inherited; but education is not inherited, it is not inheritable. It must be renewed generation after generation in each individual. The Sisyphus-stone of culture is rolled with infinite toil up the steep ascent by the fathers; it thunders instantly back, and must be rolled up again with equal agony and bloody sweat by their children."

Judged as literature, this chapter, "A Dip into the Future" is the best in the book. It maintains throughout a soberness, dignity, and elevation consonant with the subject it contemplates, suitable to the vision it beholds—the slow indeed but sure extinction of a race of men. For such is the doom that awaits the black man of the United States. But the evanescence of the Negro is not only assured, it has actually begun: the indications of the *rates* of growth, of birth, of death, and of crime, taken with other indicia, make it certain that the secular diminuendo has definitely entered upon the destined course. Even "the Negro must feel that competition is becoming sharper, that his territory is becoming narrower, that twentieth century citizenship is, like the Gospel commandment, made for those who can receive it, that he is unequal to the load cast upon him, that he is sinking beneath the burden of an honor unto which he was not born." A sombre forecast indeed. Nevertheless, "all that breathe will share their destiny. It is appointed unto men once to die." Besides, "in the upward mounting of the forms of life, there are no other stepping-stones than their dead selves. The vision of a race vanishing before its superior is not at all dispiriting, but inspiring rather. It is but a part of the increasing purpose of the ages, a forward creepng of the eternal dawn."

From beginning to end the appeal is from the individual standard to the race standard; from traditional maxims however kindly, to the warning dictates of science however stern and cold; from the relative impotence of education to the "omniprepotence of heredity"; from sentiment that feels deeply but does not see, to the imagination and the reason that depict and behold the future; from interesting academic speculation as to what might happen if things unknown were known, to the overwhelming body of ascertained fact and the doctrine of probability. Whether one does or does not agree with Professor Smith's conclusions, the candid reader will allow the book is one with which future discussions of its difficult problems will be compelled to reckon.

COLUMBIA UNIVERSITY, NEW YORK. CASSIUS J. KEYSER.

THE CONTEMPORARY DEVELOPMENT OF DIPLOMACY. By *David Jayne Hill, LL. D.* A Paper Read Before the International Congress of Arts and Science at St. Louis, on September 23, 1904.

The author of this pamphlet is an American diplomat who has done considerable service as assistant secretary of state and is now minister of the United States to Switzerland. His essay on the contemporary development of diplomacy is practically an outline of, or introduction to, a systematic presentation of diplomacy as it ought to be, the diplomacy of the future. Diplomacy has formerly been, and in some circles it is still, considered as a profession which is not bound by any principles of morality. International ethics is an ideal unknown to the diplomats of the old school, typified in Machiavelli, represented by Talleyrand, and formulated by Ancillon and Count Garden. Their view is well expressed in Henry Wotton's definition of an ambassador as "a clever man sent abroad to lie for his country." The forerunners of a new conception which would base diplomacy and all international relations upon principles of justice and fairness are mainly Gentilis and Grotius, worked out in more recent days by Bluntschli and David Dudley Field.

The civilized states of the Old World, foremost among them Germany, England, and France, recognize the change that has set in, and new methods, though inconsistent with the old ideas of sovereignty which regard the ruler of a land as the supreme authority, ranking above the law nor being bound in politics by any principles or rules of conscience, have gradually superseded the old one. The modern diplomat is no longer of the Machiavelli type, and the saying of Frederick the Great that the prince of a country is the first servant of the state, puts a new and nobler interpretation upon the old idea of sovereignty which already implies that moral maxims should also be binding for international transactions.

America, though ahead of other nations in a more modern conception of the nature of the state and less hampered by wrong ideas of sovereignty, is in some respects behind the historical development of Europe. Mr. Hill says:

"In nearly all the countries of the world,—except the United States of America,—candidates for the diplomatic service are rigorously examined before they are received, not only in international law and history, but in the laws, languages, and constitutions of other countries, and especially in commercial geography and the statistics of foreign trade. The result is, that the men who serve modern governments as diplomatic representatives are coming to have, in general, a knowledge of what is true, what is just, what is expedient, and what is right in the relations and conduct of foreign states. They constitute a valuable body of peacemakers and public advisers, whose counsel is useful because it is based on knowledge."

Diplomacy is not a new science, but it is of great importance for our international relations. "Beneath the surface of political phenomena, flows a great historical current which deserves the attention of thoughtful men. The expansive instinct of humanity changes its direction of action according to the obstacles it has to overcome. In the era of political inequality, the general aspiration was for liberty, which created in the eighteenth century a struggle for national independence; but in the constitutional era that followed, the larger human relations were revealed, and in the nineteenth century was developed the idea that modern nations are essentially interdependent. The special task of the twentieth century will be to reconcile these two great conceptions, and to unite independent states in bonds of peace, amiety, and fruitful intercourse."

The ideal of international justice has grown slowly. The state, first regarded as an ultimate and absolute authority, is now understood to rest on a moral principle. The sovereignty of its ruler has been limited by a constitution, and the constitution proves that the state itself, in order to fulfill its mission, can no longer be administrated according to the principle of absolutism, formulated by the French kings in the impudent dictum, the ultimate reason of autocracy: *car tel est notre bon plaisir.* Constitutional government has practically abolished the principle of sovereignty; but it does more. In its turn it points to a further progress of mankind implying a demand of the recognition of international morals.

Mr. Hill is not in sympathy with that morbid idealism which proclaims "the dogma that no war is just, that bloodshed is never right, and that all exercise of force is wrong. Such a doctrine owes its very possibility to the protection of institutions that would not exist for a single day, if society had not the force and determination to destroy its enemies."

A weak beginning only has been made to establish international relations upon a basis of justice, viz., the Hague Tribunal, which, however, is still looked upon with suspicion by all those who still cling to the sovereign pretensions of the state.

In the society of nations there is neither legislature, nor judiciary, nor executive, and whether it will be possible to abolish armies and navies and have them replaced by an international police who would insure the peace of the world remains to be seen.

So far the hope of abolishing standing armies and navies is only a Platonic notion, but a great advance is noticeable and the main factor in human progress toward the ideal of international ethics will be the development of an international conscience, represented by a higher respect for international ethics. Mr. Hill says:

"It will be a great advance in education, when our text-books on ethics devote their concluding chapter to international morality; for no ethical sys-

tem can be complete, either in a public or a scientific sense, which does not include in the scope of its theory the moral functions of the State and the ethics of international intercourse. When, in the schools of all civilized countries, the young are taught that moral obligation does not end with national frontiers, that states are moral entities subject to the great principles of ethics, and that treaties once freely accepted are sacred; when national history has learned to be fair and honest in its representation of other nations; a new era of human development will be opened, and diplomacy will enter upon a new period of efficiency."

LA PSYCHOLOGIE DES ROMANCIERS RUSSES DU XIX. SIÈCLE. By *Ossip-Lourié*. Paris: Félix Alcan. Pp. xv, 438. Price, 7 fr. 50.

In the introduction to his "Psychology of the Russian Novelists of the Nineteenth Century," Prof. Ossip-Lourié of the New University at Brussels furnishes a compendious review of Russian literature. His account begins with the oral expression of the confused beliefs of semi-barbarous pagans and their earliest epics. It follows the history of letters through the early monkish influences, the rise of intellectual culture with the introduction of printing, and the century and a half of further development which succeeded, until Peter the Great with the importation of Western civilization ushered in a new era. This flourishing epoch was followed by a period of imitators and critics, who gave place about 1830 to a "host of poets, novelists, dramatists, essayists, and philosophers who drew from themselves and their surroundings the fundamental elements of their works." But M. Ossip-Lourié says that in the last century the novel has had the greatest significance in Russian literature. Before Gogol the novel (or rather the story) had kept within the poetic regions of pure imagination, but since his time it has reproduced faithfully actual life with its unattractve features as well as its beauties.

Each of the great Russian novelists has risen from different social surroundings, and gives us in his work, the spirit, ideas, customs, and aspirations of his particular station. The Russian novel is made up of forces issuing from all the classes that constitute the nation, so that, taken as a whole, it gives a faithful likeness of Russian society in the nineteenth century. However, the Russian novelists are not simply portrayers of the customs of their times. They are intellectual creators, introducing into literature a new manner of thinking, and of depicting life and men. Each of them is dominated by a particular type of mind and imagination which governs both his individual and his artistic life. He sees the world through his own emotions, sentiments, and ideas, through that infinite procession of images and conceptions that he was within himself. By understanding his personality we can

better comprehend the characters that he creates, the types that he analyzes, the conditions which he presents.

The object of this present work is to study each of the Russian novelists under the different aspects of his personality and literary talent; to establish the psychological and intellectual state of each writer by the characteristic features of his life; to dissect, analyze, and define the work of each by the internal evidence of that work. After pointing out the importance of psychology in connection with literary criticism, M. Ossip-Lourié studies in turn the life and work of Gogol, Tourguenev, Gontcharov, Dostoievsky, Tolstoy, Gaichine, Tchekov, Korolenko, Maxim Gorki, and several minor writers. While showing forth the immense individual effort of these authors, he admits that no other literature has produced so many cases of pathology of the will as the Russian novel. This phenomenon is accounted for by the distinctive peculiarities of social life in Russia.

In his conclusion M. Ossip-Lourié sketches in bold outlines a psychology of Russian classes. He says that the ruling class has still remained Asiatic at heart under a veneer of civilization. The people have always been sacrificed and kept beneath the yoke of bondage. One might even say that they have been purposely inebriated in order that they might continue resistless and incapable of revolt. There is an intellectual liberal class of which a small minority strive valliantly for liberty; but the greater number have the desire without the will to oppose the elements of despotic force. They love liberty in the abstract, but they believe it is compatible with czarism, and that a social transformation can be brought about by peaceful methods. They are too indolent to do any good, and they do harm in that they prevent the progressives from acting. These last are but a small party with abundant energy and will, but no means to carry out their purposes. Many of them are found among the impetuous undergraduates of the universities.

The book closes by urging the necessity of a frank and loyal union of all the vital forces of Russia, and claims that Europe as a whole ought to consider it a moral duty to come to the aid of the slowly developing civilizing forces there.

ESQUISSE D'UNE HISTOIRE GÉNÉRALE ET COMPARÉE DES PHILOSOPHIES MÉDIÉVALLES. Par *François Picavet*, directeur-adjoint à l'Ecole pratique des Hautes Etudes. Paris: Félix Alcan. 1905. Pp. xxxvii, 366. Price, 7 fr. 50.

It is well known that Leo XIII was a very enthusiastic adherent of St. Thomas, and has done much to restore the influence of Thomism in these latter days. Undoubtedly St. Thomas has been the representative thinker of the church; yet, after all, the man who swayed not only Thomism but the entire Mediæval philosophy, was that ancient mystic, Plotinus. According

to his views, there is a sensible world, and a spiritual world (the latter commonly called *mundus intelligibilis*). This spiritual world is dominated by the principle of perfection, its highest authority being God. The soul is encompassed by the sensible world, but can attain to a union with God in states of ecstasy.

Professor Picavet, one of the most distinguished of French savants, devotes his latest work to an investigation of the Mediæval Philosophy, pointing out the significance of Plotinus. He especially indicates in the portion devoted to synchronic and comparative history, the following phases of philosophic thought pertaining to corresponding periods of time: from the Council of Nice to 529 A. D., the conflict between the neoplatonists and the Christians; from 529 to Charlemagne, the efforts to preserve as much as possible of ancient thought and civilization; from the ninth to the thirteenth centuries, the philosophical works of the Byzantines and Western Christians, the Mussulmans and Jews; from the thirteenth century to the Renaissance and the Reformation, the development of Christian thought in the Occident; from this period to the seventeenth century, the revival of ancient systems, the continued influence or restoration of Christian philosophies, the development of Protestant philosophies, and the appearance of a scientific philosophy, which up to the eighteenth and nineteenth centuries as it increased in importance, stood more and more in opposition to mediæval doctrines.

This work of Professor Picavet helps us to understand better not only the Middle Ages, but the transition from ancient to mediæval civilization, and the nature of modern society and civilization. It permits us too, to form a philosophy of religion by relying on those religions which are best known and have presented the highest and most varied forms. Texts of earlier works have been carefully studied, and the result constitutes a safe guide both for students who wish to pursue their researches further, and for those people who desire to obtain only a general idea of this interesting phase of the history of philosophy.

ZUR EINFÜHRUNG IN DIE PHILOSOPHIE DER GEGENWART. Acht Vorträge von *Alois Riehl*. Second edition. Leipsic: B. G. Teubner. 1904. Pp. 274.

Professor Riehl's "Philosophy of the Present" discusses almost all the questions which have excited popular interest in modern times, and so we need not be astonished that this treatise has reached its second edition in a very short time. His method is that of historic or critical procedure, and all systematic construction has been avoided. The book consists of eight lectures of which five are devoted to the theoretical task of philosophy, elucidating its relation to science in classical antiquity as well as in modern times. They discuss the foundation of critical philosophy, its relation to the exact sciences, the conditions of cognition, together with the most significant problems of

epistemology, monism, and the natural sciences. The last three lectures are devoted to practical questions: the problem of a systematic conception of the world, Schopenhauer and Nietzsche, and finally philosophy at present and in time to come. With special reference to the idea of personality which has been so strongly emphasized of late, Professor Riehl calls attention to Goethe's view on the subject, saying:

"No one has valued the importance of personality more highly than Goethe, who prizes it as the greatest blessing of the human race; but he also recognized and honored the super-personal, the super-human, and respected the limitations of mankind. 'The quintessence of the universe can not be comprehended in a formula; rather is it distinctly and forcibly represented in great personalities.' 'God is constantly acting in higher natures in order to help the lower to advance.' 'It is typical of great personalities to be incorporated into the national life.' And while Goethe thought highly of great personalities, he knew at the same time that even the greatest man is nothing by himself, that he does not live for himself alone. 'Whatever man may undertake and direct, the individual is not sufficient unto himself. For in fact we are all collective beings, no matter what we do. How little we have and are that in the truest sense we can call our own! We must accept and learn from those who were before us as well as from those who are with us. The main thing is for man to have a great purpose, and the determination to carry it out.'"

STUDIES IN THE GOSPEL ACCORDING TO MARK. For the Use of Classes in Secondary Schools and in the Secondary Division of the Sunday School. By *Ernest De Witt Burton*, Professor in the University of Chicago. Chicago: The University Press. 1904. Pp. xxx, 248. Price, $1.00.

The author of this book belongs to that class of theologians who recognise higher criticism. He has pointed out that the title "The Gospel According to Mark" was not written by the original author himself, and that the Appendix, Chapter XVI, Verses 9 to 20, is not really a continuation of this Gospel; but the results of higher criticism are after all withheld, for especially in the latter case the arguments are not mentioned, and no reason is given why the original has been published. Further no mention is made of any criticism that could endanger the traditional conceptions of the divinity of Christ, or the spirit that drives him into the desert, or the nature of the temptation. Buddhist parallels are nowhere mentioned. Thus the book, in spite of its advanced position in certain lines, may be considered as still representing the old conception of Christianity.

The book is attractively made up. The material is conveniently divided into sections. Illustrations help to relieve the monotony of a school book, and

a dictionary of terms is added which contains most important instruction concerning terms and names used in the Gospel; for instance such words as "Golgotha," "Son of Man," "shew bread," "Sanhedrin," "Son of David," etc. However, some very important terms are missing that stand in great need of illustration, for instance "Nazarene." The author apparently assumes that Nazarene is simply the inhabitant of Nazareth, which, according to the traditional interpretation, is identified with el-Nasira.

The seventh volume of *L'année biologique*, covering the year 1902, has just been published under the direction of Yves Delage, Member of the Institute and Professor at the Sorbonne, with the collaboration of a staff of able co-editors. The present annual is a volume of over six hundred pages and covers almost the entire range of biological science. It contains twenty chapters: I. The Cell, its Chemical Structure and its Parts, its Physiology and Functions; II. Sexual Products and Fecundation, the Origin of Embryology, Normal Fecundation, Merogony, Partial Fecundation; III. Parthenogenesis, its Conditions and Experimental Parthenogenesis; IV. Asexual Reproduction, by Division, by Budding, and by Spores; V. Ontogenesis, Tactisms, Tropisms, Functional Excitation, Ontogenetic Adaptation, Bio-mechanics; VI. Teratogenesis, Laws and Causes of the Formation of Monsters; Experimental Teratogenesis and Natural Teratogenesis; VII. Regeneration, Normal as Well as Heteromorphic; VIII. Grafting; IX. Sex and Secondary Sexual Characters; X. Metagenic Polymorphism, Metamorphism and the Alteration of Generation; XI. Latent Characters; XII. Correlations Between Organs and their Functions; XIII. Death, Senility and Immortality of Protists and Germinative Plasm; XIV. Morphology and General Physiology, Chemical Compositions of Organic Substances, Physiology of Nutrition and the Action of the Different Agents; XV. Heredity, Transmission of Characters; XVI. Variation, its Laws, its Forms, and its Causes; XVII. The Origin of Species and their Characters, Fixation of Different Kinds of Variations, New Formations, Divergences, Convergences, Phlsiological Adaptation, Physiological Species, Factors of the Formation of Species and Artificial and Natural Selection, Germinal, Sexual, etc., Segregation and Pan-mixia, Direct Influence of the Mileur, Particular Adaptations, Symbiosis, Parasitism, Mimicry, Phylogeny, the Disappearance of Species; XVIII. The Geographical Distribution of Living Forms; XIX. The Nervous System and Mental Functions, Structure and Functions of the Nerve Cells, Nervous Centers, and Sense Organs; The Nervous Cell, its Physiology and Pathology; Nerve Centers, their Structure; Cerebral Localisations, Sense Organs; Psychical Processes, Sensations, Sentiments, Ideation, Comparative Psychology; XX. General Theories.

Professor Wilhelm Windelband has delivered a memorial lecture in commemoration of the centennial return of Kant's day of death at the University of Heidelberg, and Karl Winters, the University publisher of Heidelberg, has published it in pamphlet form.* Professor Windelband epitomises in it the significance of Kant's life and thought, especially emphasising the moral tenor of his philosophy.

From Italy we have received a number of pamphlets on philosophical subjects; one by Dr. Erminio Troilo† on the doctrine of cognition in Herbert Spencer which is in reality a critical review of the philosophy of agnosticism, especially compared with Kant's critical transcendentalism.

Another pamphlet comes from Florence.‡ It is written by Dr. Guglielmo Salvadori and treats a series of problems of moral philosophy under the title *Moral Sentiment.* It discusses in the first chapter the criterion of the moral value of the ethical sentiment, in the second chapter the classification of the moral sentiment, in the third chapter the development of the moral sentiment. In the conclusion our author descants on the moral sentiment from the empirical, metaphysical, and rational point of view. The standpoint which he takes is a kind of rational eudemonism founded upon the experiences of the utilitarian school.

The third book§ is written by Dr. Roberto Ardigo and published at Turin by Bocca Brothers. It is an historical treatise of about 300 pages on the doctrine of cognition from Descartes to Kant. In the introduction he speaks of modern philosophy and its experimental methods as instanced in Galileo. Chapter I. treats of Bacon and Galileo, Chapter II., of Descartes, Leibnitz, Wolff, and Berkeley, Chapter III., of Locke, Hume, and minor contemporaries of Kant, especially Moses, Mendelssohn, Lambert, Burthogge, closing with an appreciation of the drift of modern philosophy of the character of modern thought, and the death sentence of the old philosophy as enunciated by Kant. Chapter IV. contains the *gnoseo*-logical value of the scientific revolution of modern times.

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*Immanual Kant und seine Weltanschauung.

†La Dottrina della Conoscenza pi Herbert Spencer. Bologna: Stab.

‡Saggio di uno Studio sui Sentimenti Morali. Firenze: Francesco.

§La Dottrina della Conoscenza Nei Moderni Precursori di Kant.